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Inner Estimation of the United Solution Set of Interval Linear Algebraic System

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I. Formulation of the Problem

Suppose that an interval matrix $\mathbf{A} = \{\mathbf{A}_{ij}\}$ and an interval vector $\mathbf{b} = \{\mathbf{b}_i\}$ are given. The set of all solutions $x = \{x_j\}$ of the systems $Ax = b$ for $A \in \mathbf{A}$ and $b \in \mathbf{b}$ is called the *united solution set*. It is known that the united solution set is (in general) not an interval vector. This set may have a complicated structure.

- There are many papers devoted to computing *outer estimations* for this set (see, e.g. [2, 3]).
- We present a method of finding an interval vector that is an *inner estimation* for the united solution set.

II. Kaucher's Extended Interval Arithmetic

For computing the desired inner estimation, we use the *extended interval arithmetic* proposed by Kaucher in [1].

In this formalism, basic objects are *extended intervals*, i.e., arbitrary pairs of real numbers. These pairs are denoted by $[a, b]$.

- We interpret extended intervals with $a < b$ (called *proper intervals*) as standard intervals (i.e., as sets of real numbers $[a, b]$).
- Extended intervals with $a > b$ (*improper* intervals) are used in our method as auxiliary objects in intermediate computations.

III. Main Result

We propose an algorithm that:

- starts with an extended interval matrix $\mathbf{A}$ and an extended interval vector $\mathbf{b}$, and
- returns an extended interval vector

$$\mathbf{x} = (\mathbf{x}_1, ..., \mathbf{x}_n).$$

This algorithm has the following property: If we start with proper intervals $\mathbf{A}_{ij}$ and $\mathbf{b}_i$, and all components of the resulting vector $\mathbf{x}_j$ are proper intervals, then this interval vector is an inner estimation for the united solution set of the system $\mathbf{A}x = \mathbf{b}$.

IV. Basic Idea

This algorithm is based on the following idea: It turns out that the solution of an interval algebraic system with proper intervals is related to the solution of the *dual* system $\mathbf{A}'x = \mathbf{b}'$, whose matrix $\mathbf{A}_{ij}$ is composed of the elements that are *dual* to $\mathbf{A}_{ij}$ (a dual to an extended interval $[a, b]$ is the interval $[b, a]$).

We show that an algebraic interval solution to this dual system is an inner estimation for the united solution set of the original system. So, to find the desired inner estimation, it is sufficient to find a normal solution to the algebraic system with improper interval coefficients.

For that purpose, we use an algorithm that has been originally proposed by Zyuzin [4, 5] for systems with proper interval coefficients. We show that the properties of this algorithm described in [4, 5] remain true if we use this algorithms for extended intervals.

V. Other Applications of This Idea

Other applications of this idea are described.

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