Math. 250-08, Spring 2006, Final Exam Answers, page 1.

1. $$\int_0^1 3x^2\,dx = x^3\Big|_{x=0}^{1} = 1^3 - 0^3 = 1.$$

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2. $$\begin{bmatrix}\text{Present}\\ \text{value}\end{bmatrix} = \int_0^{10} (50+100t)e^{-.05t}\,dt =$$

$$= -\frac{1}{.05}\,50e^{-.05t}\Big|_{t=0}^{10} - \frac{1}{.05}\left(100t\,e^{-.05t}\right)\Big|_{t=0}^{10}$$

$$\cancel{\$} + \frac{1}{.05}\int_0^{10} 100e^{-.05t}\,dt$$

$$= -\frac{1}{.05}\,50e^{-.05t}\Big|_{t=0}^{10} - \frac{100}{.05}\,te^{-.05t}\Big|_{t=0}^{10} - \frac{100}{(.05)^2}\,e^{-.05t}\Big|_{t=0}^{10}$$

$$= -1000(e^{-.5}-1) - 2000(10e^{-.5}) + 40000(1-e^{-.5})$$

$$= 1000 + 40000 + (-1000 - 20000 - 40000)e^{-.5}$$

$$= 41000 - 61000e^{-.5} \approx \$4001.63.$$

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3. The maximum and minimum occur either at a critical point or at ~~the~~ an end point. For the critical points:

$$f'(x) = e^{-x^2/2} + x\left(-xe^{-x^2/2}\right) = (1-x^2)e^{-x^2/2} = 0$$

where $1-x^2=0$, that is, at $x=-1$ and $x=1$. Tabulating the candidates for the maximum and minimum:

x	$f(x)$	
-10	$\approx -1.9\times10^{-21}$	
10	$\approx 1.9\times10^{-21}$	
-1	$\approx -.6065$	← minimum
1	$\approx .6065$	← maximum

4. $P(0)=6$. $P'(t)=6(.013)e^{.013t} \Rightarrow P'(0)=.078$

$P(10)=6e^{.13}\approx 6.833$; $P'(10)=6(.013)e^{.13}\approx .0888$.

These numbers mean that, in 1999 the population was 6 billion and growing at the rate of about .078 billion = 78 million per year.
In 2009, the predicted population will be 6.833 billion, and the population will be growing at the rate of about 88,800,000 per year.

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5. a) We compute $\frac{dS}{dH} = ae^{-bH} + (aH)(-be^{-bH}) = 0$

$\Leftrightarrow (a-abH)e^{-bH}=0.$

The maximum would thus have to occur where $a-abH=0$, that is, where $H=\frac{1}{b}$.

b) Increasing a increases the maximum value.

c) The maximum value is $S\big|_{H=\frac{1}{b}} = \frac{ae^{-1}}{b} = a/e$.

Thus, increasing b ~~does not change~~ decreases the maximum value.