Math 270-05, Thursday, December 1, 2005, Fifth exam answers, page 1

1 (a) It is moving fastest where the top of the graph is, between 2.5 and 3 hours after release. Its speed there is roughly 85 miles per hour.

(b) The balloon changes direction where the graph of the velocity crosses the x-axis, approximately 3.75 hours after release.

(c) The total distance travelled east is about $\int_0^{3.75} v(t)\,dt \approx$, which can be obtained by counting the number of rectangles between the graph and the x-axis between $t=0$ and $t=3.75$, then multiplying by 30. I estimate (counting portions of ~~squares or~~ rectangles), that there are 6.5 rectangles under this portion of the graph, giving $6.5 \times 30 = 195$ miles east of the starting position.

(d) (i) Again, rectangles are counted, to obtain $\approx 2(30) = 60$ miles.

(ii) $\approx 195 - .1(30) \approx 192$ miles east of the starting position.

(iii) $\approx 195 - 2.5(30) \approx 120$ miles east of the starting position.

2 (a) $\int_{-1}^{1} xe^{-x^2}dx = 0$ because xe^{-x^2} is an odd function.

(b) $\int_{-1}^{1} x^2dx = 2\int_0^1 x^2dx = \frac{2x^3}{3}\Big|_{x=0}^{x=1} = \boxed{\frac{2}{3}}$

(c) $\int_{-1}^{1} x^2+x\,dx = \int_{-1}^{1} x^2dx + \int_{-1}^{1} x\,dx = 2\int_0^1 x^2dx = \boxed{\frac{2}{3}}$

3 $\int_{-1}^{2} 2f(x)+g(x)\,dx = \int_{-1}^{1} 2f(x)+g(x)\,dx + \int_{1}^{2} 2f(x)+g(x)\,dx$

$= 2\int_{-1}^{1} f(x)dx + \int_{-1}^{1} g(x)dx + 2\int_{1}^{2} f(x)dx + \int_{1}^{2} g(x)dx$

$= 2\int_{0}^{1} g(x)dx + 2\int_{1}^{2} f(x)dx + \int_{1}^{2} g(x)dx$

$= 2(2) + 2(1) + 3 = 4+2+3 = \boxed{9}$.

4 The average value is: $\frac{1}{4-0}\int_0^4 x^2dx = \frac{1}{4}\frac{x^3}{3}\Big|_{x=0}^{4}$

$= \frac{1}{4}\frac{4^3}{3} = \boxed{\frac{16}{3}}$