Math. 270-03
Spring, 1997
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Final Examination

Thursday, May 8, 1997, 7:30–10:00AM

Instructions: This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly and explanations are given. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully. You may keep your exam sheet. Each entire problem is worth 14 points, and 2 points are "free."

1. Find:

   (a) $\frac{d}{dx}\Big(\ln\left(e^x \sin(x)\right)\Big)$ (b) $\frac{d}{dx}\left(\frac{x^2-1}{x^2+1}\right)$

   (c) $\frac{d}{dx}\sqrt{x^2+x+1}$

2. One of the functions below is a quadratic, one is a cubic, and one is a trigonometric function. Which is which? State the reasons for your conclusions.

x	$f(x)$
0.2	−0.42
0.4	−0.65
0.6	0.96
0.8	−0.15
1.2	0.84

(a)

x	$f(x)$
1.3	0.41
1.7	0.81
2.5	0.65
3.0	−0.10
3.5	−1.35

(b)

x	$f(x)$
0.5	−1.13
1.2	0.13
1.8	0.03
2.0	0.00
2.2	0.05

(c)

3. To study traffic flow along a major road, the city installs a device at the edge of the road at 4:00AM. The device counts the cars driving past, and records the total periodically. The resulting data is plotted on a graph, with time (in hours) on the horizontal axis and the number of cars on the vertical axis. The graph is shown in Figure 1; it is the graph of the function

$$C(t) = \text{Total number of cars that have passed by after } t \text{ hours.}$$

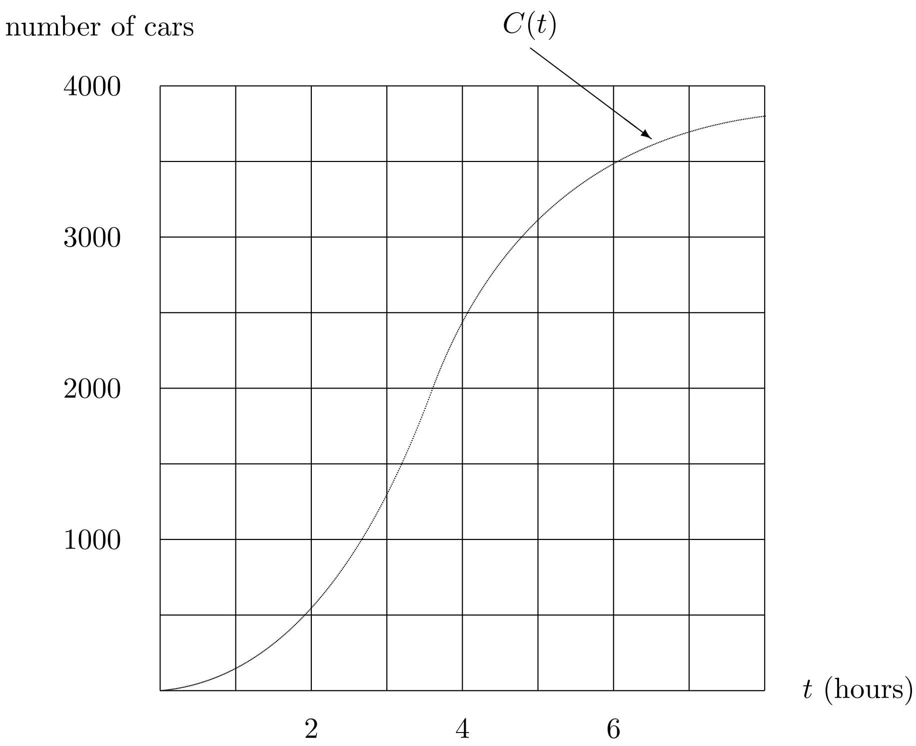

Figure 1: Traffic Along Speedway (for Problem 3)

(a) When is the traffic flow the greatest?

(b) From the graph, estimate $C'(3)$.

(c) What is the meaning of $C'(3)$? What are its units? What does the value of $C'(3)$ you obtained mean in practical terms?

4. A function defined for all x has the following properties.

- f is increasing.
- f is concave down.
- $f(5) = 2$.
- $f'(5) = 1/2$.

(a) Sketch a possible graph for $f(x)$

(b) How many zeros does $f(x)$ have and where are they located? Justify your answer.

(c) Is it possible that $f'(1) = \frac{1}{4}$? Justify your answer.

5. Find the

(a) global maximum and

(b) global minimum

of the function $f(x) = x^3 - x$ over the interval $-1 \leq x \leq 1$.

6. Eight hundred feet of fencing are available to enclose a rectangular area, one of whose sides is along a river bank (so that the fence comprises three sides of the river). What should the dimensions of the rectangle be to maximize the enclosed area?

7. Compute

$$\int_0^1 5x^4 dx$$

(a) by using the Fundamental Theorem of Calculus;

(b) approximately, by using a Riemann sum with four subdivisions.

Use the monotonicity of the integrand to compute bounds on the integral, then compare to the exact answer.