Math. 270-03
Spring, 1997
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First Examination

Thursday, Friday, February 14, 1997

Instructions: This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully. Each entire problem is worth 20 points.

1. The amount of sales tax collected in a certain city rose at a constant rate from 1960 until 1973. Between 1973 and 1982, the amount of sales tax collected increased exponentially. Between 1982 and 1986, the amount decreased gradually. From 1986 until 1990, the amount decreased exponentially. From 1990 through 1996, the amount increased exponentially, and in 1996, the amount exceeded the amount collected in 1982.

   (a) State which of the graphs in Figure 1 best approximates the graph of the sales tax collected versus time in years.

   (b) (important): Describe , individually, why each of the other graphs is not a good representation.

2. A body of mass m is falling downward with velocity v. Newton's Second Law of Motion, $F = ma$, says that the net downward force F on the body is proportional to its downward acceleration a. The net force F consists of the force F_g due to gravity, which acts downward, minus the air resistance F_r, which acts upward. The force due to gravity is mg, where g is a constant. Assume that the air resistance is proportional to the velocity of the body.

   (a) Write an expression for the net force F as a function of the velocity v.

   (b) Write a formula giving a as a function of v.

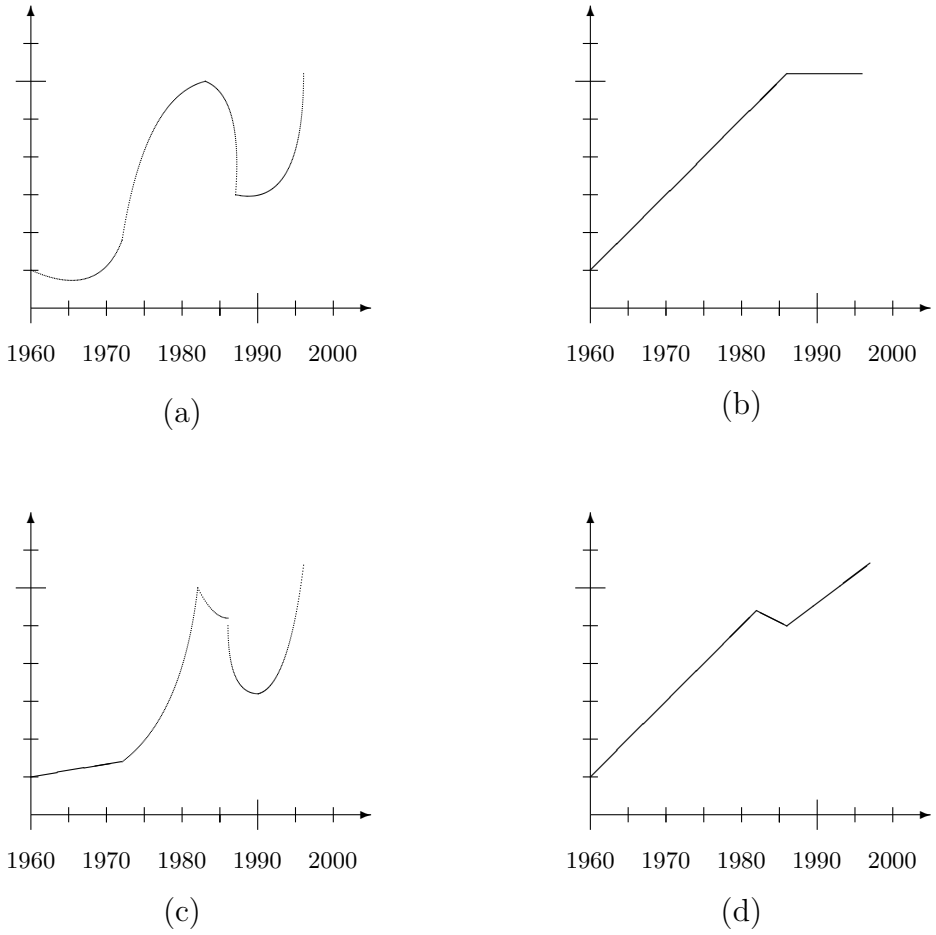

Figure 1: The graphs for problem 1

(c) Sketch a against v.

3. In the middle of the nineteenth century, a prominent London citizen remarked that "If the number of horses in this city continues to increase at the present rate, by 1950 all London streets will be 6 feet deep in horse manure." Assume that the statement was made in 1850, that there were 50,000 horses in London then, and that each horse is responsible for $1/500,000$ of a foot of horse manure in the street.

   (a) Write down an exponential model for the amount of horse manure in the street as a function of time. Call the rate constant α, and let $t = 0$ correspond to 1850.

   (b) If there were indeed 6 feet of horse manure in London in 1950, then compute the rate constant α.

   (c) With that α, compute the percentage increase per year in the number of horses in London.

   (d) What is wrong with applying such an exponential model over the period 1850–1950?

4. Each of the following functions tends to ∞ as $x \to \infty$, but they increase at different rates. Rank them from slowest to fastest; that is, list the one whose graph is on the bottom for large x first, then the one whose graph is next from the bottom, etc.

(a)	$f(x)$	$=$	$e^{0.0001x}$	(b)	$f(x)$	$=$	$\sqrt{x}$
(c)	$f(x)$	$=$	x^2	(d)	$f(x)$	$=$	$\log(x)$
(e)	$f(x)$	$=$	$x^{1/3}$	(f)	$f(x)$	$=$	x^3
(g)	$f(x)$	$=$	$x^2 + e^x$	(h)	$f(x)$	$=$	e^{x^2}

5. The earth's orbital axis changes its orientation with respect to fixed stars, or *precesses* with a period of approximately 22,000 years. A very rough model for the earth's climate assumes that the primary cause of ice ages is this precession. The minimum temperature during the last ice age was roughly 11,000 years ago, so that, according to the rough model the earth is near the maximum global temperature today. Assume that the temperature today is roughly 10° centigrade warmer than it was 11,000 years ago.

(a) Write down a trigonometric model for the deviation from average temperature of the world, as a function of time t. Take the present time to be $t = 0$.

(b) According to this model, how many degrees cooler will the earth be 1000 years from now? How many degrees cooler was the earth in the year 997 AD?

(Note: A more realistic trigonometric model is based on the Milankovich theory, in which three different periodic processes affect climate. Modern theories also include alterations to the atmosphere due to human activities.)