Math. 270-03
Spring, 1997
R. B. Kearfott

Second Examination

Friday, March 7, 1997

Instructions: This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully. Each entire problem is worth 25 points.

Table 1: Average temperature by Month

	Jan	Feb	Mar	Apr	May	Jun
°F	51.4	54.1	60.8	67.5	73.9	79.7
	Jul	Aug	Sep	Oct	Nov	Dec
°F	81.7	81.5	77.4	67.8	58.8	53.1

1. Table 1 gives average temperature by month for a certain city. Use the data to answer the following questions. (Also refer to the plot in Figure 1, where the temperature T is plotted as function of the number of months t from the beginning of the year.)
   (a) At what rate in degrees per month, approximately, is the average temperature increasing during March?
   (b) In what month does the maximum temperature occur? (Note: The maximum and minimum temperatures occur in a months in which the rate of change of temperature is equal to 0.)
   (c) In what month does the minimum temperature occur?
   (d) In what month is the temperature increasing most rapidly? What is, approximately, the maximum rate of increase?
   (e) In what month is the temperature decreasing most rapidly? What is, approximately, the maximum rate of decrease?

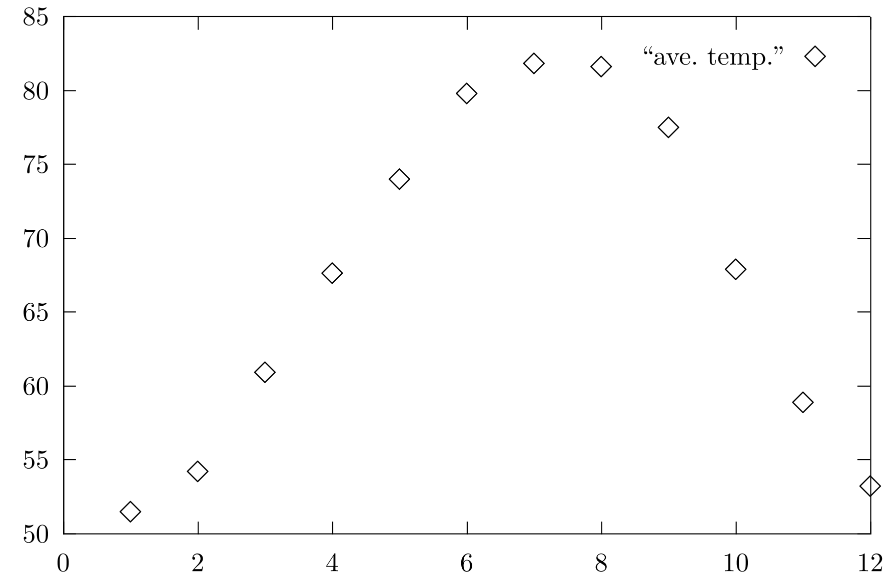

Figure 1: Plot of the data from Table 1

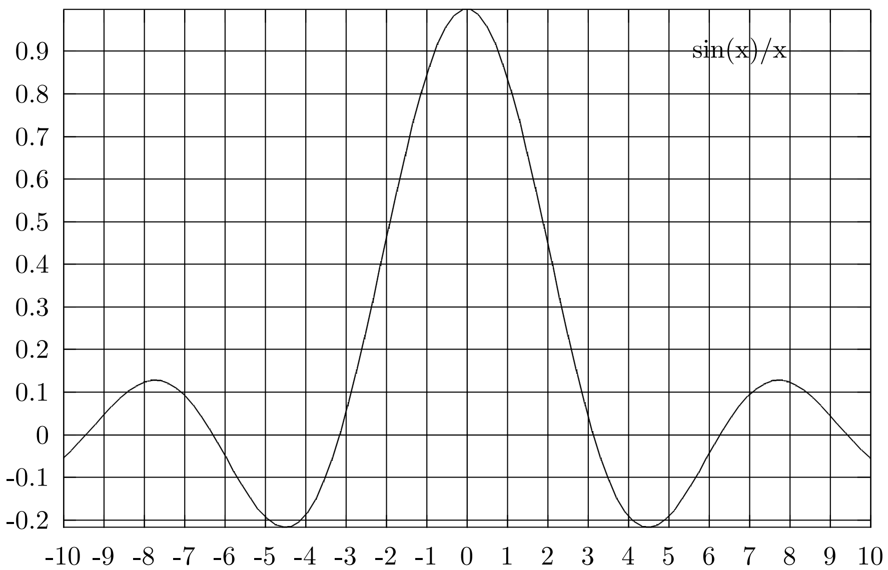

Figure 2: The graph for Problem 2

2. The graph in Figure 2 shows

$$f(x) = \begin{cases} \sin(x)/x & \text{if } x \neq 0; \\ 1 & \text{if } x = 0. \end{cases}$$

Refer to that graph to answer the following questions.

(a) Give, approximately, the values of x between -10 and 10 at which $f'(x) = 0$.

(b) Give, approximately, a value of x between -10 and 10 for which $f'(x)$ is maximum. What can you say about f'' there? What is this maximum f'?

(c) Give, approximately, a value of x between -10 and 10 for which $f'(x)$ is minimum. What is this minimum f'?

(d) Give approximate ranges of x between -10 and 10 for which $f''(x) > 0$.

(e) Give approximate ranges of x between -10 and 10 for which $f''(x) < 0$.

3. Use the definition of the derivative as a limit of a difference quotient, that is, use

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

to compute the derivative function $f'(x)$ to $f(x) = x^3$.

4. A person with a certain liver disease first exhibits larger and larger concentrations of certain enzymes (called SGOT and SGPT) in the blood. As the disease progressses, the concentration of these enzymes drops, first to the predisease level and eventually to zero (when almost all of the liver cells have died). Monitoring the levels of these enzymes allows doctors to track the progress of a patient with this disease. If $C = f(t)$ is the concentration of the enzymes in the blood as a function of time:

(a) Sketch a possible graph of $C = f(t)$.

(b) Mark on the graph the intervals where $f' > 0$ and where $f' < 0$.

(c) What does $f'(t)$ represent, in practical terms?