

Fourth Examination
Friday, April 18, 1997

Instructions: This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully. Keep your exam sheet.

1. (48 points) Find $f'(x)$, for $f(x)$ as follows:

$$\begin{array}{ll} \text{(a)} & f(x) = 3x^2 - 2x + 1 \\ \text{(c)} & f(x) = \frac{\sin(x)}{x} \\ \text{(e)} & f(x) = \arctan(1 + x) \end{array} \quad \begin{array}{ll} \text{(b)} & f(x) = e^{x^2} \\ \text{(d)} & f(x) = \sqrt{1 - x^2} \\ \text{(f)} & f(x) = x^2 e^x \end{array}$$

2. (20 points) Find dy/dx at $x = 0, y = 1$, if $(x - 1)^2 + y^2 = 2$.
3. (32 points) The acceleration g in meters per second per second due to gravity, at a distance of r kilometers from the center of the earth, is given approximately by

$$g(r) = \frac{4.002 \times 10^8}{r^2}. \quad (1)$$

- (a) Find $\frac{dg}{dr}$.
- (b) Use a linear approximation to $g(r)$ at the surface of the earth ($r = 6400$ kilometers) to form a table of approximations to $g(r)$ at 200, 400, and 600 kilometers above the earth.
- (c) Compute $g(7000)$ directly from equation (1), and compare with the corresponding value from the linearization? Is the actual value less than or greater than the value given in the linearization? How could you tell this from the graph of $g(r)$? Illustrate with a graph.
- (d) Based on the value $\frac{dg}{dr}$ at $r = 6400$, is it safe to say that g is constant within 5 kilometers (about the height of Mount Everest) of the surface of the earth?