Math. 270-10
Spring, 2017
R. B. Kearfott

Final Examination

Friday, May 5, 2017, 2:00PM to 4:30PM

Instructions: This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly and explained. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully.

1. Write down the equation for the tangent line to the graph of $f(x) = \arctan(x)$ at $x = 1/2$.
2. Suppose f is a function with $\lim_{h\to 0} \dfrac{f(2+h)-3}{h} = 4$. Write down $f(2)$ and $f'(2)$.
3. If $x^3 + y^2 = 1$, find $\frac{dy}{dx}$ in terms of x and y.
4. An aircraft if flying horizontally at a constant height of 4000 ft above a fixed observation point. (See the figure on the board.) At a certain instant the angle of elevation θ is 45° and decreasing, and the speed of the aircraft is 200 mi/h.
   (a) How fast it θ decreasing at this instant? Express the result in units of deg/sec.
   (b) How fast is the distance between the aircraft and the observation point changing at this instant? Express the result in units of ft/sec. Use 1 mile = 5280 ft.
5. The side of a square is estimated to be 5 meters, with an error of ± 0.01 meter. Use differentials to estimate the error in the calculated area.
6. Find all critical points of $f(x) = x^6 - 6x^4$. Classify each critical point as corresponding to a relative maximum, relative minimum, or neither.
7. Suppose $s(t) = t^6 - 6t^4$ describes the position of a particle moving along a coordinate line, where s is in feet and t is in seconds.
   (a) Find the velocity and acceleration functions.
   (b) Find the position, velocity, speed, and acceleration at time $t = 1$.
   (c) At what times is the particle stopped?
8. Use L'Hopital's rule to evaluate $\lim_{x\to 0} \dfrac{e^{2x}-1}{\sin(3x)}$.
9. Calculate a single number equal to $\displaystyle\int_0^{\pi/2} \frac{\cos(x)}{1+\sin^2(x)}\,dx$.
10. Calculate a number representing the difference between the area bounded by the portion of the graph of $f(x) = x^3$ above the x-axis and the axis and the area bounded by the portion of the graph of $f(x) = x^3$ below the x-axis and the x-axis, over the interval $[-1, 3]$.
    (*Hint: The quantity is a definite integral.*)