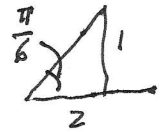
Math. 270-10, Spring, 2017, Final exam answers, page 1

① $y = f(\tfrac{1}{2}) + f'(\tfrac{1}{2})(x-\tfrac{1}{2})$. $f(\tfrac{1}{2}) = \arctan(\tfrac{1}{2}) = \tfrac{\pi}{6}$



$f'(x) = \dfrac{1}{1+x^2}$, so $f'(\tfrac{1}{2}) = \dfrac{1}{1+\frac{1}{4}} = \dfrac{1}{\frac{5}{4}} = \dfrac{4}{5}$.

Thus, the equation is: $\boxed{y = \frac{\pi}{6} + \frac{4}{5}(x-\frac{1}{2})}$.

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② $\boxed{f(2) = 3, \text{ and } f'(2) = 4}$

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③ $3x^2 + 2y\dfrac{dy}{dx} = 0$, so $\boxed{\dfrac{dy}{dx} = -\dfrac{3x^2}{2y}}$

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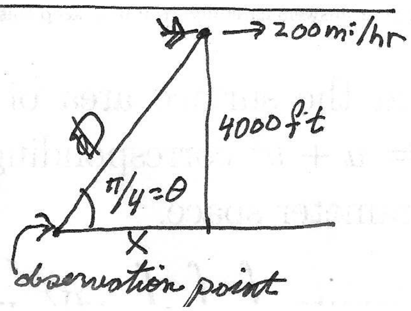
④ $\dfrac{200\text{ mi}}{\text{hr}} = \dfrac{200(5280)}{3600}$ ft/sec

$= 293.\overline{3}$ ft./sec



$\dfrac{x}{4000} = \cot(\theta)$

$\dfrac{dx}{dt} = -4000\csc^2(\theta)\dfrac{d\theta}{dt} = 293.\overline{3}$, $\csc^2(\frac{\pi}{4}) = 2$

so $\dfrac{d\theta}{dt} = \dfrac{-293.\overline{3}}{8000} \approx -.03\overline{6}$ radians/sec $\approx (-.03\overline{6})\left(\dfrac{180}{\pi}\right)$ degrees/sec

$\approx \boxed{2.1 \text{ degrees/sec.}}$ ⓐ

ⓑ $D = \sqrt{4000^2+x^2}$, so $\dfrac{dD}{dt} = \dfrac{x}{\sqrt{4000^2+x^2}}\dfrac{dx}{dt}$ in ft/sec $= \dfrac{4000\sqrt{2}}{(\sqrt{3})4000}\,293.\overline{3}$,

so $\dfrac{dD}{dt} = \sqrt{\dfrac{2}{3}}\,(293.\overline{3}) \approx \boxed{239.5 \text{ ft/sec}}$ ⓑ

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⑤ $A(s) = s^2$ $dA = 2s\,ds \approx 2(5)(.01) = .1$ meter $\approx \dfrac{.1}{25} \times 100\% = 0.4\%$

∴ The error is approximately $\boxed{\pm 10\text{ cm}^2}$

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⑥ $f'(x) = 6x^5 - 24x^3 = 6x^3(x^2-4) = 0$ at $x=0, x=2, x=-2$.

$f''(x) = 30x^4 - 72x^2 = 6x^2(5x^2-12)$

$f''(2) > 0$, so $\boxed{x=2 \text{ corresponds to a local minimum}}$

$f''(-2) > 0$, so $\boxed{x=-2 \text{ corresponds to a local minimum}}$

f' changes from positive to negative at 0

so $\boxed{x=0 \text{ corresponds to a local maximum}}$.

(7) (a) $\boxed{\begin{array}{l} v(t) = s'(t) = 6t^5 - 24t^3. \\ a(t) = v'(t) = 30t^4 - 72t^2 \end{array}}$.

(b) $s(1) = 1 - 6 = \boxed{-5}$, $v(1) = 6 - 24 = \boxed{-18}$, (speed) = 18, $a(1) = 30 - 72 = \boxed{-42}$

(c) The particle is stopped where $v(t) = 0$, namely, at $\boxed{t = -2,\ t = 0, \text{ and } t = 2.}$

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(8) $\displaystyle \lim_{x \to 0} \frac{e^{2x} - 1}{\sin(3x)} = \lim_{x \to 0} \frac{2e^{2x}}{3\cos(3x)} = \boxed{\frac{2}{3}}$

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(9) $\displaystyle \int_0^{\pi/2} \frac{\cos(x)}{1 + \sin^2(x)}\, dx$

$u = \sin(x)$
$du = \cos(x)\, dx$

$$\int \frac{\cos(x)}{1 + \sin^2(x)}\, dx = \int \frac{du}{1 + u^2} = \arctan(u) + C = \arctan(\sin(x)) + C,$$

so $$\int_0^{\pi/2} \frac{\cos(x)}{1 + \sin^2(x)}\, dx = \arctan(\sin(x)) \Big|_{x=0}^{\pi/2} = \arctan(1) - \arctan(0) = \boxed{\pi/4}$$

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(10) $$\int_0^3 x^3\, dx + \int_{-1}^0 x^3\, dx = \int_{-1}^3 x^3\, dx = \frac{x^4}{4}\Big|_{-1}^{3} = \frac{1}{4}(81 - 1) = \boxed{\frac{80}{4} = 20}$$
