

Math. 270-10, Spring, 2017, First exam answers, page 1

① (a) $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x^2 + 1} = \boxed{0}$ (b) $\lim_{x \rightarrow 1} \frac{x - 1}{x^2 - 1} = \lim_{x \rightarrow 1} \frac{1}{x+1} = \boxed{\frac{1}{2}}$

(c) $\lim_{x \rightarrow +\infty} \frac{x^3 + x + 1}{3x^3 - x - 1} = \boxed{\frac{1}{3}}$

(d) $\lim_{x \rightarrow -\infty} \frac{x^3 + x + 1}{3x^3 - x - 1} = \boxed{\frac{1}{3}}$

(e) $\lim_{x \rightarrow \infty} \frac{x^3 + x + 1}{3x^3 - x - 1} = \lim_{x \rightarrow \infty} \frac{x + \frac{1}{x^2} + \frac{1}{x^3}}{3 - \frac{1}{x} - \frac{1}{x^2}} = \boxed{+\infty}$

(f) $\lim_{x \rightarrow -\infty} \frac{x^3 + x + 1}{3x^3 - x - 1} = \boxed{-\infty}$

② (a) $\lim_{x \rightarrow 1} \sqrt{f(x)} = \boxed{N} \lim_{x \rightarrow 1} f(x) = \sqrt{4} = \boxed{2}$

(b) $\lim_{x \rightarrow 1} \frac{1}{g(f(x))} = \lim_{y \rightarrow 4} \frac{1}{g(y)} = \boxed{0}$

(c) $\lim_{x \rightarrow 4} h(g(x)) = \lim_{y \rightarrow \infty} h(y) = \boxed{10}$

③ (a) $\lim_{x \rightarrow 0} \frac{\sin(2x)}{\sin(3x)} = \lim_{x \rightarrow 0} \left\{ \frac{2x \frac{\sin(2x)}{2x}}{3x \frac{\sin(3x)}{3x}} \right\} = \frac{2}{3} \cdot \frac{1}{1} = \boxed{\frac{2}{3}}$

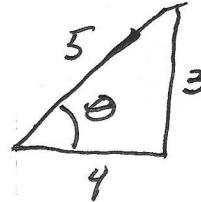
(b) $\lim_{x \rightarrow \pi/2} \tan(x)$ does not exist,

(c) $\lim_{x \rightarrow \pi/2} \tan^2(x) = +\infty$

(d) $\lim_{x \rightarrow \infty} \tan^{-1}(x) = \pi/2$

(e) $\lim_{x \rightarrow -\infty} \tan^{-1}(x) = -\pi/2$

④ $\cos(\sin^{-1}(\frac{3}{5})) = \boxed{\frac{4}{5}}$



⑤ a) $\ln(e^2) = 2\ln(e) = \boxed{2}$

b) $e^{\ln(3^2+3+1)} = 3^2 + 3 + 1 = 9 + 3 + 1 = \boxed{13}$

c) $\ln\sqrt{e^{3/5}} = \ln((e^{3/5})^{1/2}) = \ln(e^{3/10}) = \boxed{\frac{3}{10}}$