

①  $(x_0, y_0) = (1, f(1)) = (1, 1)$ .

$m = f'(x_0); f'(x) = 2x; f'(x_0) = 2(1) = 2 = m$

$y = y_0 + m(x - x_0) \quad \therefore y = 1 + 2(x - 1) \text{ or } y = 2x - 1$

② (a)  $f'(x) = 2x - 5$

(b)  $f(x) = (x^3 - x)(x+1)^2$ , so

$f'(x) = (3x^2 - 1)(x+1)^2 + (x^3 - x)(-2)(x+1)^{-3}$ ,

or  $f'(x) = \frac{(3x^2 - 1)(x+1) - 2x^3 + 2x}{(x+1)^3}$

$= \frac{3x^3 - x + 3x^2 - 1 - 2x^3 + 2x}{(x+1)^3}$

$= \frac{x^3 + 3x^2 + x - 1}{(x+1)^3}$

(c)  $f'(x) = x\pi \cos \pi x + \sin \pi x$

(d)  $f'(x) = 2xe^{-x} - x^2e^{-x}$

(e)  $f'(x) = \frac{1}{(x-x_0)^2 + 1}$

(f)  $f'(x) = [x^{-1} \sin(x)]' = -x^{-2} \sin(x) + x^{-1} \cos(x)$

$= \frac{-\sin(x) + x \cos(x)}{x^2}$

(g)  $f'(x) = \ln(x) + x \left(\frac{1}{x}\right)' = \ln(x) + 1$

(h)  $f'(x) = \frac{1}{\ln(10)} \cdot \frac{1}{(x^3 + x^2)} (3x^2 + 2x)$

$= \frac{1}{\ln(10)} \frac{x(3x+2)}{x(x^2+x)} = \frac{1}{\ln(10)} \frac{3x+2}{x^2+x}$

$$(3) \ln(f) = \frac{1}{5} \ln(x^2 + 2x + 1) - \frac{3}{4} \ln(x^3 - x)$$

$$\text{so } \frac{1}{f(x)} f'(x) = \frac{1}{5(x^2 + 2x + 1)} (2x + 2) - \frac{3}{4} \cdot \frac{1}{x^3 - x} (3x^2 - 1)$$
$$= \frac{2}{5(x+1)} - \frac{3(3x^2 - 1)}{4(x^3 - x)}$$

$$\text{so } f'(x) = \frac{(x^3 - x)^{3/4}}{(x^2 + 2x + 1)^{1/5}} \left[ \frac{2}{5(x+1)} - \frac{3(3x^2 - 1)}{4(x^3 - x)} \right]$$

(4) (a) The units are barrels/year

(b) It would be negative, since the amount of recoverable petroleum generally decreases (barring technical advances)

$$(c) P(11) \approx P(10) + \frac{dP}{dt} \Big|_{T=10} (11 - 10)$$

$$\approx 10,000 - 1000 = 9000$$