

① (a) $\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = \lim_{x \rightarrow 0} \frac{e^x}{1} = \boxed{1}$

(b) $\lim_{x \rightarrow \infty} \frac{x}{e^x} = \lim_{x \rightarrow \infty} \frac{1}{e^x} = 0$

(c) $\lim_{x \rightarrow \infty} x^2 e^{-x} = \lim_{x \rightarrow \infty} \frac{x^2}{e^x} = \lim_{x \rightarrow \infty} \frac{2x}{e^x} = \lim_{x \rightarrow \infty} \frac{2}{e^x} = 0$

② The y-intercept is $R(0) = \frac{3}{2}$. The x-intercept is where $3+x=0 \Rightarrow \boxed{x=-3}$. The only vertical asymptote is where $2-x=0$, at $\boxed{x=2}$. Any extrema occur where

$$R' = [(3+x)(2-x)^{-1}]' = (2-x)^{-1} - (2-x)^{-2}(-1)(3+x)$$

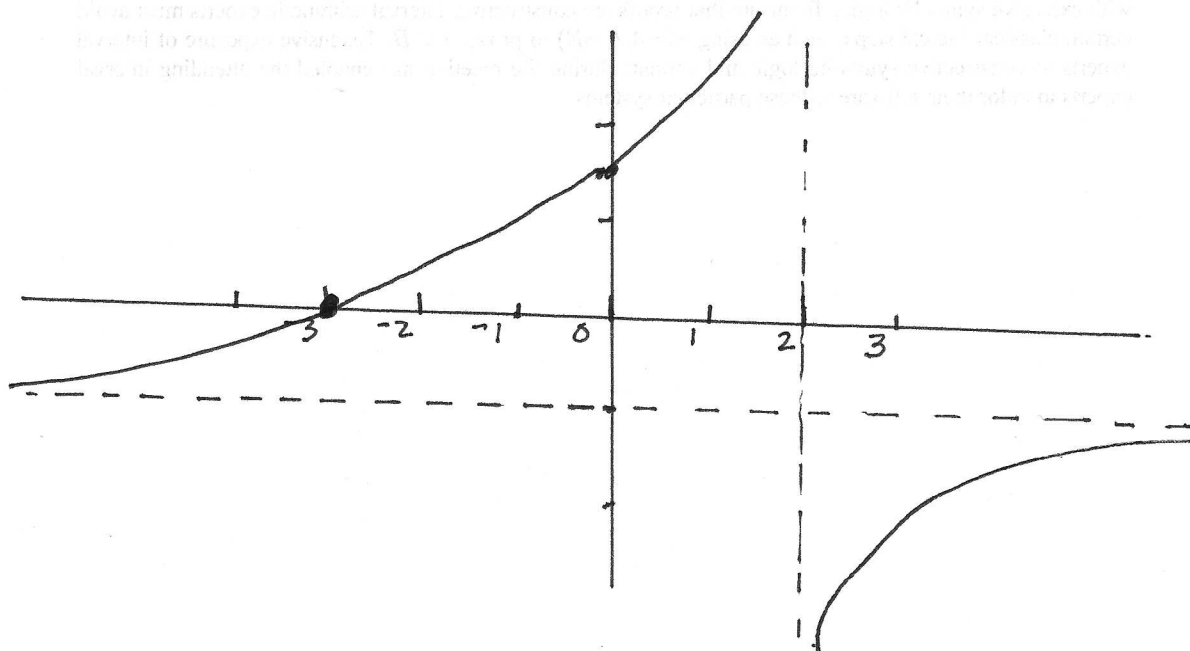
$$= \frac{(2-x) + (3+x)}{(2-x)^2} = \frac{5}{(2-x)^2} = 0, \text{ not possible, so there}$$

are no local extrema. Concave up and concave down regions are determined by

$$R'' = [5(2-x)^{-2}]' = -10(2-x)^{-3}(-1) = \frac{10}{(2-x)^3}$$

$\frac{10}{(2-x)^3} > 0$ for $x < 2$, and $\frac{10}{(2-x)^3} < 0$ for $x > 2$. Thus, the graph is concave up for $x < 2$ and concave down for $x > 2$. $\lim_{x \rightarrow \infty} \frac{3+x}{2-x} = 1 = \lim_{x \rightarrow -\infty} \frac{3+x}{2-x}$, so there is only

one horizontal asymptote, at $y=1$



③ Critical points of f occur at

$f'(x) = 3x^2 - 3 = 3(x^2 - 1) = 0$, at
 $x = -1$ and $x = 1$. $x = -1$ is the only
 critical point in the interval
 being considered. Thus, the
 absolute maximum is 3 (at $x = -1$), and the
 absolute minimum is -1 (at $x = -2$).

x	$f(x)$
-2	-1
0	1
-1	3

④ The average rate of change is $\frac{f(2) - f(0)}{2 - 0} = \frac{2 - 0}{2 - 0} = 1$.

$f'(x) = 3x^2 - 3 = 1$ where $3x^2 = 4$, $x^2 = \frac{4}{3} \Rightarrow x = \pm \frac{2}{\sqrt{3}}$.

Thus, $C = \frac{2}{\sqrt{3}}$.