Math. 301-01
Spring, 2007
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Final Examination

Thursday, May 10, 2007

Instructions: This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly and explained. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully. You may keep this exam sheet.

1. (20 points) If $F'(x) = f(x)$ is as in the graph in figure 1, and $F(0) = 0$, then draw a corresponding graph of $F(x)$. Be sure to label the axes on your graph.

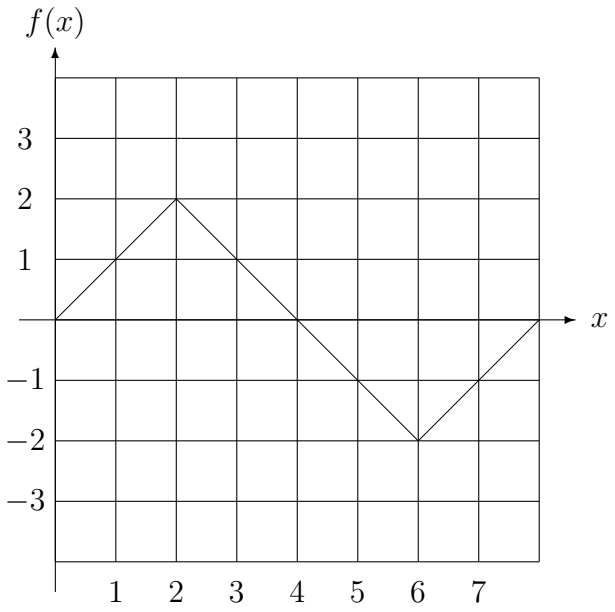

Figure 1: The figure for problem 1

2. (5 points apiece) Compute the following indefinite and definite integrals; show your work.

a. $\displaystyle\int xe^{-x}dx$ b. $\displaystyle\int_0^{\infty} xe^{-x}dx$ c. $\displaystyle\int_0^{\pi/2} \cos(x)e^{\sin(x)}dx$

d. $\displaystyle\frac{d}{dx}\int_{t=2x}^{t=x^2} e^{\sin(t)}dt$ e. $\displaystyle\int_1^2 \frac{x+2}{(x+3)(x+4)}dx$ f. $\displaystyle\int_0^2 \sqrt{4-x^2}dx$

3. (15 points) Compute the volume of the solid obtained by rotating the portion of the curve $y = x^2$ between $x = 0$ and $x = 1$ about the line $y = 2$.

4. (10 points) Compute the degree-2 Taylor polynomial for $f(x) = e^x$ expanded about $x_0 = \ln(2)$.

5. (25 points) Suppose we wish to approximate $f(x) = \cos(2x)$ by a degree-2 polynomial.

   (a) Write down the degree-2 Taylor polynomial $T_2(x)$ for f centered at $x = 0$.

   (b) Using your calculator, write down 8 digits for $f(.01745329252)$ (.01745329252 is the radian measure of approximately 1°.)

   (c) Compute $T_2(.01745329252)$ to at least 8 digits and compute $T_2(.01745329252) - f(.01745329252)$.

   (d) Compute a bound on the error in $f(.01745329252) - T_2(.01745329252)$ and compare your bound to the value for $f(.01745329252) - T_2(.01745329252)$ you computed in part 5c.

   Note: You may enter $2 \times .01745329252$ *into your calculator as* $\pi/90$.