Math. 301-01
Spring, 2007
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Fourth Examination

Tuesday, April 17, 2007

Instructions: This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly and explained. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully. You may keep this exam sheet. Each entire problem is worth 25 points.

1. After measuring the duration of many telephone calls, the telephone company found their data was well-approximated by the density function $p(x) = 0.5e^{-0.5x}$, where x is the duration of a call in minutes.
   (a) What percentage of calls last between 1 and 2 minutes?
   (b) What percentage of calls last 1 minute or less?
   (c) What percentage of calls last 3 minutes or more?
   (d) Find the cumulative distribution function.
2. Find the mean duration of a telephone call and the median duration of a telephone call in problem 1.
3. Use your knowledge of geometric series to
   (a) write down
   $$.6 + .06 + .006 + .0006 + \cdots$$
   as a fraction.
   (b) Do the same for
   $$.1 + .01 + .001 + .0001 + \cdots$$
4. Determine whether or not the following series converge absolutely, converge conditionally, or diverge. Explain the theorems that you used in each case.
   (a) $\displaystyle\sum_{n=0}^{\infty} n2^{-n}$ (b) $\displaystyle\sum_{n=0}^{\infty} \frac{n!}{(2n)!}$ (c) $\displaystyle\sum_{n=2}^{\infty} (-1)^{n-1} \frac{1}{\ln(n)}$