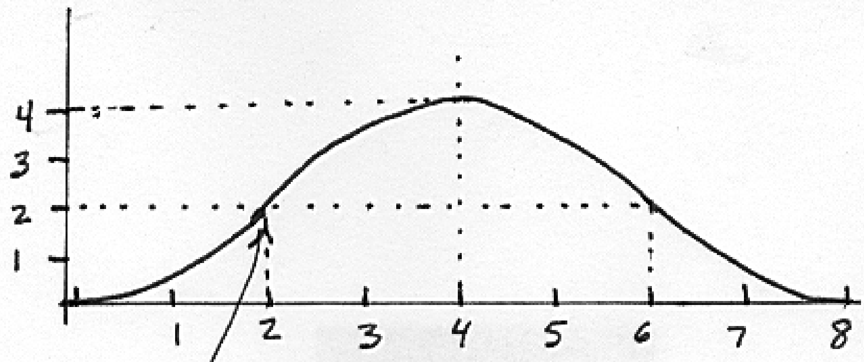
1



2 a $\int \underbrace{x}_{u}\underbrace{e^{-x}}_{v'}dx = -xe^{-x} + \int e^{-x}dx = \boxed{-xe^{-x} - e^{-x} + C}$

b $\int_0^\infty xe^{-x}dx = \lim_{M\to\infty} \left[-xe^{-x} - e^{-x}\right]\Big|_{x=0}^{M} = 1.$

c $\int_0^{\pi/2} \cos(x)e^{\sin(x)}dx$ $\boxed{\begin{matrix} u = \sin(x) \\ du = \cos(x)dx \end{matrix}}$

$= \int_{u=0}^{u=1} e^u du = e^u\Big|_{u=0}^{1} = \boxed{e - 1 \approx 1.71828}$

d $\frac{d}{dx}\int_{t=2x}^{t=x^2} e^{\sin(t)}dt = \boxed{e^{(\sin(x^2))}(2x) - e^{\sin(2x)} \cdot 2}$

(e) $$\frac{x+2}{(x+3)(x+4)} = \frac{A}{x+3} + \frac{B}{x+4}$$

$$(x+2) = A(x+4) + B(x+3) = (A+B)(x) + (4A+3B).$$

$$\Rightarrow \left\{\begin{array}{l} A+B=1 \\ 4A+3B=2 \end{array}\right\} \Leftrightarrow \left\{\begin{array}{l} +3A+3B=3 \\ 4A+3B=2 \end{array}\right\} \Rightarrow \boxed{A=-1} \Rightarrow \cancel{3B=-2} \Rightarrow \boxed{\cancel{B=\tfrac{-2}{3}}}$$

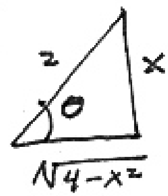
$$\Rightarrow \boxed{B=2} \Rightarrow \frac{x+2}{(x+3)(x+4)} = \frac{-1}{x+3} + \frac{2}{x+4}$$

$$\Rightarrow \int_1^2 \frac{x+2}{(x+3)(x+4)}\,dx = \int_1^2 \frac{-1}{x+3} + \frac{2}{x+4}\,dx = \Big[-\ln|x+3| + 2\ln|x+4|\Big]_{x=1}^{2}$$

$$= [-\ln 5 + 2\ln 6 + \ln(4) - 2\ln 5] = -3\ln 5 + 2\ln 6 + \ln 4$$

$$= \ln\left[\frac{6^2 \cdot 4}{5^3}\right] = \ln\left[\frac{144}{125}\right] = \ln(1.152) \approx 0.141499$$

(f) $$\int_0^2 \sqrt{4-x^2}\,dx$$

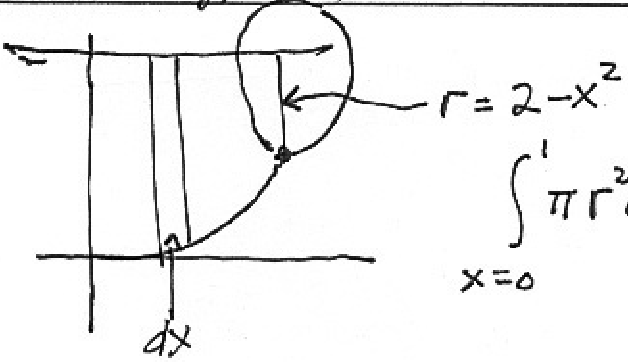


$$x = 2\sin\theta$$
$$dx = 2\cos(\theta)\,d\theta$$
$$\sqrt{4-x^2} = 2\cos(\theta)$$

$$= \int_{\theta=\arcsin(0)}^{\arcsin(1)} 2\cos(\theta)(2\cos(\theta)\,d\theta)$$

$$= 4\int_0^{\pi/2} \cos^2\theta\,d\theta = 4\int_0^{\pi/2} \frac{1-\cos(2\theta)}{2}\,d\theta = 4\left(\frac{1}{2}\right)\left(\frac{\pi}{2}\right) = \boxed{\pi}.$$

(3)



$$\int_{x=0}^{1} \pi r^2\,dx = \int_{x=0}^{1} \pi\,(2-x^2)^2\,dx$$

$$= \int_{x=0}^{1} \pi(4-4x^2+x^4)\,dx = \pi\left[4x - \frac{4}{3}x^3 + \frac{1}{5}x^5\right]\Big|_{x=0}^{1}$$

$$= \pi\left[4 - \frac{4}{3} + \frac{1}{5}\right] = \boxed{\pi\left[\frac{43}{15}\right] \approx 9.0059}$$

---

(4) $f(\ln(2)) = f'(\ln(2)) = f''(\ln(2)) = 2$, so the Taylor polynomial is:

$$T_2(x) = 2 + 2(x-\ln(2)) + \frac{2}{2}(x-\ln(2))^2$$

$$= 2 + 2(x-\ln(2)) + (x-\ln(2))^2$$

---

(5) (a) $T_2(x) = 1 - \dfrac{(2x)^2}{2!}$

(b) $f(\pi/90) \approx .99939082702$

(c) $T_2(\pi/90) \approx .999390765$

$f(\pi/90) - T_2(\pi/90) \approx 6.2\times 10^{-8}$

(d) Since the Taylor series for f at $\pi/180$ is an alternating series whose terms have absolute values that are decreasing, the error is bounded by the first term left off, namely by

$\dfrac{(2\pi/180)^4}{4!} \approx 6.18\times 10^{-8}$. The discrepancy is due to the fact that there is roundoff error in subtraction, we only listed 8 digits, and the error is on the order of 10^{-8}.