

$$\textcircled{1} \textcircled{a} \int_1^2 .5 e^{-.5x} dx = -e^{-.5x} \Big|_{x=1}^2 = \frac{1}{\sqrt{e}} - \frac{1}{e} \approx .23865 \approx 24\%$$

$$\textcircled{b} \int_0^1 .5 e^{-.5x} dx = -e^{-.5x} \Big|_{x=0}^1 = 1 - \frac{1}{\sqrt{e}} \approx .39347 \approx 39\%$$

$$\textcircled{c} \int_3^{\infty} .5 e^{-.5x} dx = -e^{-.5x} \Big|_3^{\infty} = \frac{1}{e^{3/2}} \approx .22313 \approx 22\%$$

$$\textcircled{d} F(x) = \int_0^x .5 e^{-.5t} dt = -e^{-.5t} \Big|_{t=0}^x = \boxed{1 - e^{-.5x}}$$

$$\begin{aligned} \textcircled{2} \mu &= \int_0^{\infty} .5x e^{-.5x} dx = -x e^{-.5x} \Big|_{x=0}^{\infty} + \int_0^{\infty} e^{-.5x} dx \\ &= \frac{-1}{.5} e^{-.5x} \Big|_{x=0}^{\infty} = \boxed{2 \text{ minutes}} \leftarrow \text{mean} \end{aligned}$$

$$[\text{median}] = M, \text{ where } \int_0^M .5 e^{-.5x} dx = \frac{1}{2}$$

$$\text{That is, } -e^{-.5x} \Big|_{x=0}^M = \frac{1}{2} \Leftrightarrow 1 - e^{-.5M} = \frac{1}{2}$$

$$\Leftrightarrow e^{-.5M} = \frac{1}{2} \Leftrightarrow -.5M = \ln\left(\frac{1}{2}\right) \Leftrightarrow$$

$$M = \frac{-1}{.5} \ln\left(\frac{1}{2}\right) = 2 \ln 2 \approx 1.386 \text{ min.}$$

③ (a) $.6 + .06 + \dots = \sum_{n=0}^{\infty} .6 (10^{-n}) = .6 \sum_{n=0}^{\infty} 10^{-n} = .6 \left(\frac{1}{1 - \frac{1}{10}} \right)$
 $= \frac{6}{10} \left(\frac{10}{9} \right) = \frac{6}{9} = \boxed{\frac{2}{3}}$

(b) $.1 + .01 + \dots = (.1) \sum_{n=0}^{\infty} 10^{-n} = \frac{1}{10} \left(\frac{1}{1 - \frac{1}{10}} \right) = \frac{1}{10} \left(\frac{10}{9} \right) = \boxed{\frac{1}{9}}$

④ (a) We use the ratio test: $\left| \frac{a_{n+1}}{a_n} \right| = \frac{(n+1) 2^{-(n+1)}}{n 2^{-n}} = \left(\frac{n+1}{n} \right) \cdot \frac{1}{2}$
 $\rightarrow \frac{1}{2} < 1$ as $n \rightarrow \infty$. Thus, the series converges absolutely.

(b) Again, we use the ratio test:

$$\frac{(n+1)!}{(2(n+1))!} = \frac{(n+1)(n!) (2n)!}{(2n+2)(2n+1)(2n)! (n!)}$$
$$\frac{n!}{(2n)!} = \frac{(n+1)}{(2n+2)(2n+1)} \rightarrow 0 < 1 \text{ as } n \rightarrow \infty. \text{ Thus,}$$

the series converges absolutely.

④ The series is an alternating series, and $\frac{1}{\ln(n)}$ decreases monotonically to zero, so, by the alternating series test, the series converges. However,

$$\frac{1}{\ln(n)} \geq \frac{1}{n} \text{ and } \sum_{n=2}^{\infty} \frac{1}{n} \text{ diverges, so } \sum_{n=2}^{\infty} (-1)^{n-1} \frac{1}{\ln(n)}$$

does not converge absolutely; that is,

$$\sum_{n=2}^{\infty} (-1)^{n-1} \left(\frac{1}{\ln(n)} \right) \text{ only converges conditionally.}$$