Math. 301-01
Fall, 1999
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Final Examination
Wednesday, December 8, 1999

Instructions: This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly and explained. This exam is open book, open notes, and computer-on. You may leave after handing in your exam paper, but be sure to check your answers carefully. Each entire problem is worth 14 points, and two points are free.

1. If $f(x)$ is as in Figure 1, then compute $\int_{x=1}^{4} f(x)dx$.

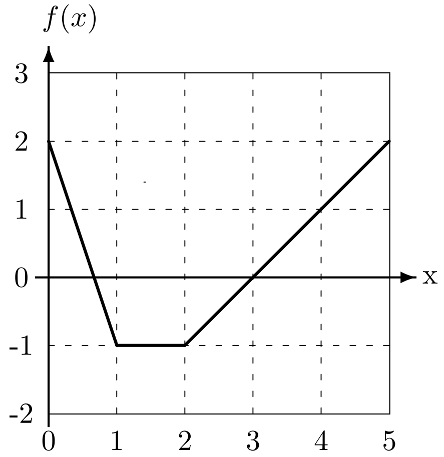

Figure 1: The function f for Problem 1

2. Simplify the following expressions. You may use a computer program, but state that you have done so if you do. In any case, state the principle behind finding each of the indefinite integrals, and show all work.

$$\text{(a)}\ \frac{d}{dx}\int_{t=0}^{x}\frac{\sin(t^{1999})}{t}dt \qquad \text{(b)}\ \int x^3\cos(2x)dx \qquad \text{(c)}\ \int_0^1 xe^{x^2}dx$$

3. Use the trapezoid rule with 10 and 100 subintervals to approximate the definite integral in Problem 2. Compute numerical values for the actual error, based on the exact analytical value obtained from solution of Problem 2.
   (a) Are the approximations overestimates or underestimates to the actual value? Based on the function, why should this be so?
   (b) What is the ratio of the two errors? What significance does this have?

4. State which of the following improper integrals converge. Where possible, give exact values for the improper integrals that converge. If it is not possible to give exact values for a convergent integral, show how you know it converges.

$$\text{(a)} \int_{x=\pi}^{\infty} e^{-x}\frac{\sin(x)}{x}dx \qquad \text{(b)} \int_{0}^{1} \frac{e^x}{x^2}dx \qquad \text{(c)} \int_{1}^{\infty} \frac{e^x}{x^2}dx$$

Hint: For part (c), first determine what happens to e^x/x^2 *as* $x \to \infty$.

5. A cylindrical storage tank with diameter 50 feet and height 25 feet is initially empty, and is to be pumped full of gasoline. If gasoline weighs 42 pounds per cubic foot, how much work, in foot pounds, must be expended to fill the tank from the bottom?

6. Consider the function $f(x) = x\cos(x)$.
   (a) Compute the degree 5 Taylor polynomial $P_5(x)$ for $f(x)$ centered at $x = 0$.
   (b) Write down the error term. *Hint: The degree-5 Taylor polynomial is also the degree-6 Taylor polynomial. The error term can thus be of degree 7.*
   (c) Suppose $P_5(x)$ is used to approximate $f(x)$ for $x \in [0, 0.1]$.
      i. Is $P_5(x)$ an overestimate, underestimate, or neither on this interval? Why?
      ii. Use your error term to give as bound on the possible value of $f(x) - P_5(x)$ for $x \in [0, 0.1]$.
   (d) Compute $f(0.1)$, $P_5(0.1)$, and the actual error $f(0.1) - P_5(0.1)$, to at least ten significant figures, using your calculator or computer. Compare this actual error to the error bound you obtained in part 6(c)ii.

7. A hot steel rod at 500° C is placed in a large tub of ice water at 0° C. How long will it take before the steel rod is less than 60° C, if it is observed to cool to 400° C in half an hour?