Math. 301-01
Fall, 1999
R. B. Kearfott

Third Examination
Tuesday, November 9, 1999

Instructions: This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly and explained. This exam is open book, open notes, and computer-on. You may leave after handing in your exam paper, but be sure to check your answers carefully. Each part of each problem is worth 16 points, and 4 points are "free".

1. Consider the function $f(x) = xe^x$.
   (a) Write down the degree-6 Taylor polynomial $T_6(x)$, centered at $x = 0$, for f.
   (b) Write down the error term. (*Hint: The error term for f can be obtained by multiplying the error term for e^x by x.*)
   (c) Suppose $T_6(x)$ is to be used to approximate xe^x for $x \in [0, 0.2]$.
      i. Is the polynomial an overestimate, an underestimate, or neither for the actual value of xe^x? Carefully explain why.
      ii. Use your error term to give a bound on the possible value of $|f(x) - T_6(x)|$ for $x \in [0, 0.2]$.
   (d) Compute $f(0.2)$, $T_6(0.2)$, and the actual error $f(0.2) - T_6(0.2)$. (That is, compute a numerical approximation to this, using your calculator or the computer. Compare this actual error to the error bound you obtained in part 1(c)ii.

2. Consider the power series
$$\sum_{k=0}^{\infty} 5^k (x-1)^k.$$
   (a) What is this series' radius of convergence?
   (b) Based on that radius of convergence, give an interval of the form $[a, b]$ such that $x \in [a, b]$ implies the limit $\sum_{k=0}^{\infty} 5^k (x-1)^k$ exists.