

Fourth Examination
Friday, December 3, 1999

Instructions: This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly and explained. This exam is open book, open notes, and computer-on. You may leave after handing in your exam paper, but be sure to check your answers carefully. Each entire problem (excluding the extra credit) is worth 33 points, and one point is free.

1. Consider the differential equation $y' = xy$.
 - (a) Sketch the slope field for $-3 \leq x \leq 3$ and $-3 \leq y \leq 3$.
 - (b) Sketch the curve corresponding to the initial condition $y(1) = 1$.
 - (c) Solve the initial value problem $y' = xy$, $y(1) = 1$ analytically by using separation of variables. Show all your computations.
2. Consider Euler's method to solve approximately the initial value problem $y' = xy$, $y(1) = 1$, for $1 \leq x \leq 2$.
 - (a) Compute an approximate solution $y(1.1)$ with stepsizes $h = 0.1$, $h = 0.05$, and $h = 0.025$. If doing by hand, show your work, and round to six significant digits. If doing it with Matlab, print each value.
 - (b) Compute the ratios of errors corresponding to these h 's, using the exact solution you computed in the first problem. Are the two ratios approximately the same? Are they approximately the same as the ratios of stepsizes?
 - (c) (*15 points extra credit added to any exam score*) Use Matlab to find approximate solutions to $y(2)$, using $h = 1/10$, $h = 1/100$, and $h = 1/1000$. Plot your exact solution and three approximate solutions on the same coordinate axes. Hand in your plots with a discussion of what they mean.
3. A hurricane floods a freshwater marsh with salt water at 3.5% salt. The total volume of the marsh is 800 million cubic feet. Suppose that a bayou flows into the marsh at the rate of 0.288 million cubic feet per day, and a similar bayou flows out of the marsh at the other end. At that rate, how many days will it take for the salt concentration to reach less than 0.5%?