

**Fourth Examination**  
*Friday, November 17, 2000*

**Instructions:** This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly and explained. This exam is closed book, but you may use your calculator and the computers. You may leave after handing in your exam paper, but be sure to check your answers carefully. Give exact values, rather than numerical approximations, unless the problem asks for a numerical approximation. Each entire problem is worth 20 points.

1. Write down the terms of degree five for the Taylor series for  $f(x) = \cos(x)$  centered at  $x = \pi/2$ .
2. Write down the radius of convergence of the series

$$\sum_{n=0}^{\infty} \frac{(x-1)^n}{n3^n}$$

3. Write down the terms of degree 5 or less for the Taylor series for

$$f(x) = \frac{\sin(x^3)}{x^2}$$

centered at  $x = 0$ . Show all work, or explain in detail how you obtained the terms.

4. Racketeers dump one ton (2000 pounds) of soluble pollutant into a lake at the beginning of every week. Clean water from a bayou flowing into the lake mixes with the polluted water, and the mixed water flows out of the lake. Through this process, 75% of the pollutant in the lake is removed every week.
  - (a) How many pounds of pollutant are in the lake before the beginning of week 10?
  - (b) How many pounds of pollutant are in the lake in the limit as time passes, right before the racketeers dump the next load?
5. Compute a bound on the error if  $f(x) = \log(1+x)$  is approximated by a Taylor polynomial of degree 4 centered at  $x = 0$ , over the interval  $[-0.1, 0.1]$ .