Fifth Examination
Wednesday, November 30, 2011

Instructions: This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly and explained. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully. Each entire problem is worth 25 points. You may keep this exam sheet.

1. Write down the degree 6 Taylor polynomial for $f(x) = \sin(x)$ about the point $a = \pi/2$. Show your work.

2. Write down the first four non-zero terms of the Taylor series expanded about $x = 0$ for the function
$$f(x) = \frac{1}{1 + x^2}.$$
Show your work. (Note: There is more than one correct way of doing this one easily.)

3. Use Taylor polynomials to determine which of the following functions is largest and which is smallest for $x > 0$ and $|x|$ small.

   (a) $f(x) = \dfrac{\sin(x)}{x} - 1$; (b) $g(x) = \dfrac{e^x - 1}{x} - 1$; (c) $h(x) = \dfrac{\log(1 + x) - x}{x^2} + \dfrac{1}{2}$.

   Show your work.

4. How many terms must you take to approximate $f(x) = \cos(x)$ to within 10^{-6} in the interval defined by $|x| \leq 0.1$ by a Taylor polynomial centered at $x = 0$? Show your work.
   (*Hint:* $|\sin(x)| \leq 0.1$ *for* $|x| \leq 0.1$. *Also, there is more than one way of doing this problem correctly.*)