1 (a) $$\int_0^{\pi} \overset{u}{x}\,\underbrace{\sin 2x\,dx}_{dv} = -\frac{1}{2}x\cos 2x\Big|_0^{\pi} + \frac{1}{2}\int_0^{\pi}\cos(2x)\,dx$$

$$= -\frac{1}{2}\pi\cos 2\pi + 0\cos 0 + 0 = \cancel{\boxed{0}}\;.\;\boxed{= -\frac{\pi}{2}}$$

---

(b) $$\int \frac{dx}{x^2+3x+2} = \int \frac{dx}{(x+1)(x+2)}$$

$$= \int \frac{dx}{x+1} - \int \frac{dx}{x+2}$$

$$= \ln|x+1| - \ln|x+2| + C$$

$$= \ln\left|\frac{x+1}{x+2}\right| + C$$

$$\frac{1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}.$$

$$1 = A(x+2) + B(x+1)$$

$$= (A+B)x + (2A+B)$$

$$A+B=0 \Rightarrow B=-A$$

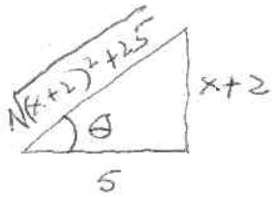
$$2A-A=1 \Rightarrow A=1$$

---

(c) $$\int \frac{dx}{x^2+4x+29} = \int \frac{dx}{(x+2)^2+25}$$

$$= \int \frac{5\sec^2\theta}{25\sec^2\theta}\,d\theta = \frac{1}{5}\int d\theta$$

$$= \frac{1}{5}\theta + C = \boxed{\frac{1}{5}\arctan\left(\frac{x+2}{5}\right) + C}$$




$$x+2 = 5\tan\theta$$

$$dx = 5\sec^2\theta\,d\theta$$

$$(x+2)^2+25 = 25\sec^2\theta$$

---

(d) $$\int_{-1}^{1}\sqrt{1-x^2}\,dx = \int_{-\pi/2}^{\pi/2}\cos^2(\theta)\,d\theta$$

$$= \int_{-\pi/2}^{\pi/2}\frac{1+\cos(2\theta)}{2}\,d\theta = \int_{-\pi/2}^{\pi/2}\frac{d\theta}{2} = \boxed{\frac{\pi}{2}}$$


$$x = \sin(\theta)$$

$$dx = \cos(\theta)\,d\theta$$

$$\sqrt{1-x^2} = \cos\theta$$

2 (a) Simpson's Rule is $\frac{b-a}{6}\left(f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right)$

$$= \frac{1}{3}\left((0^2+3\cdot 0+2)+4(1^2+3(1)+2)+(2^2+3(2)+2)\right)$$

$$= \frac{1}{3}(2+\cancel{20}\,24+12) = \frac{1}{3}(38) = \boxed{\frac{38}{3}}$$

(b) $$\int_0^2 x^2+3x+2\,dx = \left[\frac{1}{3}x^3+\frac{3}{2}x^2+2x\right]\Bigg|_{x=0}^{2}$$

$$= \left[\frac{8}{3}+6+4\right]-0 = \boxed{\frac{38}{3}}$$

(c) The two values are exactly equal. This is because Simpson's rule integrates polynomials of degree 3 or less exactly.

3 (a) $$\int_0^\infty \frac{1}{(x+2)^2}\,dx = \lim_{M\to\infty}\left[\frac{-1}{x+2}\right]\Bigg|_0^M = \cancel{\tfrac{1}{2}}$$

$$= \lim_{M\to\infty}\left[\frac{-1}{M+2}+\frac{1}{2}\right] = \boxed{\frac{1}{2}}.$$

(b) $$\int_{-2}^{2} \frac{dx}{(x+2)^2} = \lim_{\varepsilon\to 0^+}\left[\frac{-1}{x+2}\right]\Bigg|_{-2+\varepsilon}^{2} = \lim_{\varepsilon\to 0^+}\left[-\frac{1}{4}+\frac{1}{\varepsilon}\right].$$

The limit does not exist, since $\frac{1}{\varepsilon}\to\infty$ as $\varepsilon\to 0^+$.

3c $\displaystyle\int_1^\infty \frac{dx}{\sqrt{x}} = \lim_{M\to\infty}\left[2x^{1/2}\right]\Big|_{x=1}^{M} = \lim_{M\to\infty}\left[2\sqrt{M}-2\right]$

does not exist, since $2\sqrt{M}-2 \to \infty$ as $M \to \infty$.

3d $\displaystyle\int_0^1 \frac{dx}{\sqrt{x}} = \lim_{\varepsilon\to 0^+}\left[2\sqrt{x}\right]\Big|_{x=\varepsilon}^{1} = \lim_{\varepsilon\to 0^+}\left(2-2\sqrt{\varepsilon}\right) = \boxed{2}$