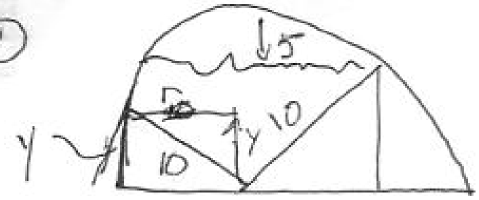
1



$r = \sqrt{100 - y^2}$

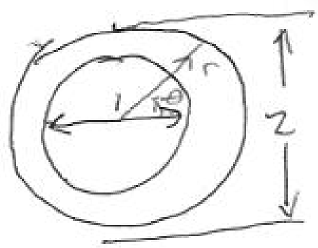
[volume] $= \pi r^2 \Delta y$

The total volume inside the tank filled to 10−5=5 ft. deep is

$$\int_{y=0}^{5} \pi \left(\sqrt{10^2 - y^2}\right)^2 dy = \pi \int_0^5 (100 - y^2)\, dy$$

$$= \pi \left(100y - \frac{y^3}{3}\right)\Big|_{y=0}^{5} = \pi\left(500 - \frac{125}{3}\right) \approx 1439.9 \text{ cubit feet.}$$

2



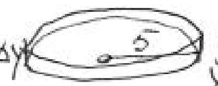
$1/2 \le r \le 1$

$0 \le \theta \le 2\pi$

(see problems 16-19, p.414 of the text)

3

The pressure y feet down in the oil, $0 \le y \le 5$ is $50y$. Thus, the force on a piece of surface y ft. down and of



height Δy is [pressure][area] $= 50y(\pi(10)\Delta y) = 500\pi y \Delta y$.

The total force on the vertical side is thus

$$\int_{y=0}^{5} 500\pi y\, dy = 500\pi \frac{y^2}{2}\Big|_{y=0}^{5} = 250(\pi)(25) \approx 19{,}635 \text{ lb.} \approx 10 \text{ tons}$$

The force on the bottom is simply

[pressure][area] $= 50(5)(\pi \cdot 5^2) \approx 19{,}635$ lb,

the same as on the vertical surface, in this case.

(4) [center] = $\dfrac{[\text{first moment}]}{[\text{weight}]} = \dfrac{\int_0^1 x\delta(x)\,dx}{\int_0^1 \delta(x)\,dx}$.

$$\int_0^1 x\delta(x)\,dx = \int_0^1 x - xe^{-x}\,dx = \frac{x^2}{2}\Big|_{x=0}^{1} - \left[-xe^{-x}\Big|_{x=0}^{1} + \int_0^1 e^{-x}\,dx\right]$$

$$= \frac{x^2}{2}\Big|_0^1 + xe^{-x}\Big|_0^1 + e^{-x}\Big|_0^1 = \frac{1}{2} + \frac{1}{e} + \frac{1}{e} - 1 = \frac{-1}{2} + \frac{2}{e}$$

$$\int_0^1 \delta(x)\,dx = \int_0^1 (1-e^{-x})\,dx = [x+e^{-x}]\Big|_{x=0}^{1} = 1 + \frac{1}{e} - 1 = \frac{1}{e}.$$

Thus, the center of mass is at:

$$\bar{x} = \frac{\frac{2}{e} - \frac{1}{2}}{\frac{1}{e}} = 2 - \frac{e}{2} \approx 0.6409.$$

(Note $\bar{x}$ is to the right of center, since most of the mass is to the right of center.)