June 13, 2005

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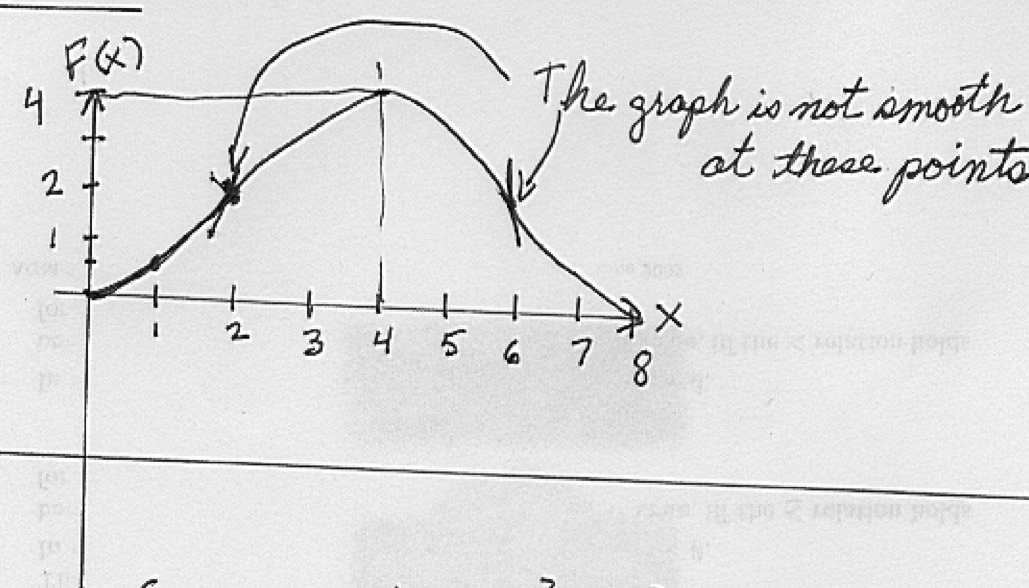

2 a $\int x^2+2x+1\,dx = \frac{x^3}{3}+x^2+x+C.$

b $\int (x+1)^2\,dx = \frac{1}{3}(x+1)^3+C = \frac{x^3}{3}+x^2+x+\tilde{C}.$

c $\int 2x\sin x^2\,dx = \int \sin(u)\,du$ ← Integration by ~~parts~~: substitution. $u=x^2$, $du=2x\,dx$

$= -\cos(x^2)+C$

d $\int t^2e^t\,dt = t^2e^t - \int 2te^t\,dt$

$= t^2e^t - \left[2te^t - \int 2e^t\,dt\right]$

$= t^2e^t - 2te^t + 2e^t + C$

e $\int x\ln x\,dx = \frac{x^2}{2}\ln x - \int \frac{x^2}{2}\cdot\frac{1}{x}\,dx$

$= \frac{x^2}{2}\ln x - \frac{1}{2}\int x\,dx = \frac{x^2}{2}\ln x - \frac{x^2}{4} + C$

2f $\dfrac{1}{(x+1)(x+2)} = \dfrac{A}{x+1} + \dfrac{B}{x+2} \Rightarrow 1 = A(x+2) + B(x+1)$

Plugging in -2 for ~~A~~ x gives $B = -1$, and plugging in -1 for x gives $A = 1$. Thus,

$$\int \frac{dx}{(x+1)(x+2)} = \int \frac{1}{x+1}\,dx - \int \frac{1}{x+2}\,dx = \ln|x+1| - \ln|x+2| + C$$

$$= \ln\left|\frac{x+1}{x+2}\right| + C$$

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3 $$[\text{Area}] = \int_{x=0}^{2} (x^2+2x+1)\,dx = \frac{1}{3}(x+1)^3\Big|_{x=0}^{2} = \frac{1}{3}[27-1] = \boxed{\frac{26}{3}}$$

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4 Let $s(t)$ be the distance the jet has travelled at time t, where $s(0)=0$. The length of the runway is ~~$s(30)$~~, $s\left(\frac{30}{3600}\right)$ where $s'(0)=0$.

We have $$s''(t) = \frac{200-0}{\frac{30\,\text{sec}}{3600\,\text{sec/hr}}} = \frac{200\,\text{mph}}{\frac{1}{120}\,\text{hr}} = 24{,}000\,\text{mph/hr}$$

so

$$s'(t) = 24{,}000t + 0$$

so

$$s(t) = 12{,}000t^2 + 0$$

$$s\left(\frac{30}{3600}\right) = 12{,}000\left(\frac{1}{120}\right)^2 \approx .833\overline{3}\ \text{mi} \approx 4400\,\text{ft}.$$