1 (a) $\displaystyle\int_{x=1}^{\infty} \frac{dx}{x^{1.001}}$ converges, and can be computed

explicitly: $\displaystyle\int_{x=1}^{\infty} \frac{dx}{x^{1.001}} = \lim_{M\to\infty} \int_{x=1}^{M} x^{-1.001}\,dx$

$\displaystyle = \lim_{M\to\infty} \frac{1}{-.001} x^{-.001} \Big|_{x=1}^{\infty} = \lim_{M\to\infty} \left[-M^{-.001}+1\right]\cdot 1000$

$= \boxed{1000}$

(b) $\displaystyle\int_{x=0}^{1} \frac{dx}{x^{1.001}}$ diverges, since it is of the form $\displaystyle\int_{x=0}^{a} \frac{dx}{x^{p}}$ with $p>1$.

(c) $\displaystyle\int_0^{\infty} \frac{e^{-x}}{\sqrt{x}}\,dx = \int_0^1 \frac{e^{-x}}{\sqrt{x}}\,dx + \int_1^{\infty} \frac{e^{-x}}{\sqrt{x}}\,dx$

The first integral converges, since $\dfrac{e^{-x}}{\sqrt{x}} = \dfrac{e^{-x}}{x^{1/2}} \le \dfrac{1}{x^{1/2}}$ for $x>0$ and $\displaystyle\int_0^1 \frac{dx}{x^{1/2}}$ converges.

The second integral converges, since $\dfrac{e^{-x}}{\sqrt{x}} \le e^{-x}$ for $x>1$ and $\displaystyle\int_1^{\infty} e^{-x}\,dx$ converges.

The integral cannot be evaluated without resorting to numerical techniques

② $$[\text{Volume}] = \int_{y=0}^{5} \pi\left[\frac{2}{5}y\right]^2 dy = \int_{y=0}^{5} \frac{4}{25}\pi y^2\,dy$$

$$= \frac{4}{25}\pi \frac{y^3}{3}\Big|_{y=0}^{5} = \frac{4}{25}\pi\frac{(125)}{3} = \boxed{\frac{20\pi}{3} \text{ cubic centimeters}}$$

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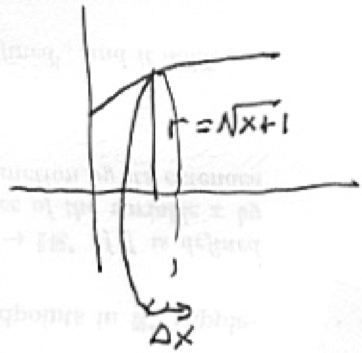
③ $$[\text{Volume}] = \int_{x=0}^{1} \pi\left(\sqrt{x+1}\right)^2 dx$$

$$= \int_{x=0}^{1} \pi(x+1)\,dx = \pi\left[\frac{x^2}{2}+x\right]\Big|_{x=0}^{1}$$

$$= \pi\left[\frac{1}{2}+1\right] = \boxed{\frac{3}{2}\pi}.$$



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④ $f(x) = x^{3/2}$, so $f'(x) = \frac{3}{2}x^{1/2}$, so $\sqrt{1+(f'(x))^2} = \sqrt{1+\frac{9}{4}x}$ for $x>0$. Therefore,

$$[\text{Length}] = \int_{x=0}^{1} \sqrt{1+9/4\,x}\;dx$$

$$\boxed{\begin{aligned} u &= 1+9/4\,x \\ du &= \frac{9}{4}dx \\ dx &= \frac{4}{9}du \end{aligned}}$$

$$= \frac{4}{9}\int_{u=1}^{13/4} u^{1/2}\,du = \frac{4}{9}\left(\frac{2}{3}u^{3/2}\right)\Big|_{u=1}^{13/4}$$

$$= \left(\frac{4}{9}\right)\left(\frac{2}{3}\right)\left[\left(\frac{13}{4}\right)^{3/2} - 1\right] = \cancel{\frac{8}{27}}$$

$$= \frac{8}{27}\left[\left(\frac{13}{4}\right)^{3/2} - 1\right] \approx 1.43971$$

The total mass of the object is

$$m = \int_0^{10} e^{-x}\,dx = -e^{-x}\Big|_{x=0}^{10} = 1-e^{-10}\ \text{kg}.$$

The first moment of the object is:

$$(\text{moment}) = \int_0^{10} x e^{-x}\,dx = -xe^{-x}\Big|_{x=0}^{10} + \int_0^{10} e^{-x}\,dx$$

$$= -10e^{-10} + 1 - e^{-10} = 1 - 11e^{-10}$$

so

$$\bar{x} = \frac{1-11e^{-10}}{1-e^{-10}} \approx .9995$$