① We form a table:

n	$f^{(n)}(x)$	$f^{(n)}(\pi/2)$
0	$\cos(x)$	0
1	$-\sin(x)$	-1
2	$-\cos(x)$	0
3	$\sin(x)$	1
4	$\cos(x)$	0
5	$-\sin(x)$	-1

Thus, the degree 5 Taylor polynomial is:

$$P_5(x) = \frac{-(x-\frac{\pi}{2})}{1!} + \frac{(x-\frac{\pi}{2})^3}{3!} - \frac{(x-\frac{\pi}{2})^5}{5!}$$

Alternately, we see $\cos(x) = -\sin(x+\pi/2)$, so we can see $P_5(x)$ is correct.

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② ⓐ We derive the Taylor polynomial from the Taylor polynomial for $\sin(x)$:

$$\cancel{P_4(x)}\ \sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}\cos(c)$$

so $$\frac{\sin(x)}{x} = \boxed{1 - \frac{x^2}{3!} + \frac{x^4}{5!}} - \frac{x^6}{7!}\cos(c)$$

ⓑ The Taylor polynomial is an overestimate, since the error term is negative for $-\pi/2 \le x \le \pi/2$.

3 The error term in the taylor polynomial of degree $2n$ is $\frac{(-1)^{n+1} x^{2n+2}}{(2n+3)!}\cos(c)$. Its absolute value is bounded by $\frac{1}{(2n+3)!}$ for $|x| \le 1$. We form a table to find the n that works:

n	$\frac{1}{(2n+3)!}$
0	$1/3! \approx .1667$
1	$1/5! \approx .008$
2	$1/7! \approx 1.9 \times 10^{-4}$
3	$1/9! \approx 2.7 \times 10^{-6}$
4	$1/11! \approx 2.5 \times 10^{-8}$
5	$1/13! \approx 1.6 \times 10^{-10}$
6	$1/15! \approx 7.6 \times 10^{-13}$

Thus, the Taylor polynomial of degree $2(6) = \boxed{12}$ will do.

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4 The Taylor ~~polynomial~~ series for $\ln(1+u)$ is:

$$u - \frac{u^2}{2} + \frac{u^3}{3} - \frac{u^4}{4} + \cdots$$ so the series for $\ln(1+3x)$ is:

$$3x - \frac{(3x)^2}{2} + \frac{(3x)^3}{3} - \frac{(3x)^4}{4} + \cdots + \frac{(-1)^{n+1}(3x)^n}{n} + \cdots$$

The ratio $\frac{|A_{n+1}|}{|A_n|}$ is: $\dfrac{\frac{|3x|^{n+1}}{n+1}}{\frac{|3x|^n}{n}} = \frac{3^{n+1}|x|^{n+1}}{3^n |x|^n}\left(\frac{n}{n+1}\right)$

$$= 3|x|\left(\frac{n}{n+1}\right) \to 3|x| \text{ as } n \to \infty.$$

$3|x| < 1$ for $|x| < \frac{1}{3}$. Therefore, the radius of convergence is $\boxed{\frac{1}{3}}$