

First Examination
Wednesday, September 9, 2009

Instructions: This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly and explained. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully. Each entire problem is worth 20 points.

1. Classify the graphs of each of the following equations as (i) a cylinder (parabolic, hyperbolic, circular, or otherwise), (ii) a paraboloid, (iii) a hyperboloid of one sheet, (iv) a hyperboloid of two sheets, (v) a plane, or (vi) an ellipsoid (egg-shaped region).

$$(a) \quad \frac{x^2}{4} + y^2 + \frac{z^2}{9} = 1, \quad (b) \quad x + y + z = 1, \quad (c) \quad x^2 + y^2 = 1,$$

$$(d) \quad \frac{x^2}{4} + \frac{y^2}{9} - z = 0, \quad (e) \quad x^2 + y^2 - z^2 = 1.$$

2. Sketch a rough graph of each of the surfaces in (a), (b), and (c) of problem 1. Label with the numerical value the points on each axis (if any) where each of the surfaces intersects the axis.
3. Write down an expression for the linear function whose graph contains the point $(0, 0, 1)$, with slope in the x -direction equal to -1 , and with slope in the y -direction equal to -1 .
4. Sketch the $z = 0$, $z = 1$, and $z = 4$ contours of the function $f(x, y) = x^2/4 + y^2/9$. Label these contours on your sketch. State what types of curves these contours are.
5. Does the following function have a limit at $(x, y) = (0, 0)$? Give a detailed reason why or why not.

$$f(x, y) = \frac{x^3 + y^3}{x + y}.$$