

**Fourth Examination**  
*Thursday, October 22, 2009*

**Instructions:** This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly and explained. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully. Each part of each problem is worth 25 points. You may keep this question sheet.

1. Find and classify all critical points of

$$f(x, y) = x^2 + 2yx - 4x + 2y^2 - 6y + 5.$$

2. (a) Use Lagrange multipliers to find the global minimum of  $f(x, y)$  subject to  $x + y = 1$ , where  $f$  is as in Problem 1.  
(b) Find the minimum of  $f(x, y)$  subject to  $x + y = 1$  by solving for one of the variables in the constraint, plugging into the objective function, and minimizing the resulting function of one variable. Verify that you obtain the same answer as in part (a).
3. A garden is to be built on a large plot of land, but the garden must be fenced to keep out the raccoons. Fencing costs \$1 per meter for the sides and \$2 per meter for the front and back of the garden. What should the dimensions of the garden be to maximize the area of the garden, if a total of \$250 is available for fencing?