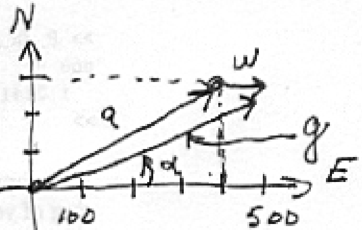
Math. 302-03, Fall, 2009, Second Exam Answers, 2009/09/23 (page 1)

(1) $\vec{v} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$. $\vec{u} = \frac{1}{\|\vec{v}\|}\vec{v}$. $\|\vec{v}\| = \sqrt{\vec{v}\cdot\vec{v}} = \sqrt{1+1+1}$

$$\boxed{\therefore \ \vec{u} = \begin{bmatrix} 1/\sqrt{3} \\ -1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}}$$

---

(2) Let w be the vector corresponding to the wind, and let a be the vector corresponding to the airspeed, and let g be the vector corresponding to movement of the plane with respect to the ground. Then



$$g = a + w.$$

$$a = 500\begin{bmatrix} 4/5 \\ 3/5 \end{bmatrix} = \begin{bmatrix} 400 \\ 300 \end{bmatrix}; \ w = \begin{bmatrix} 80 \\ 0 \end{bmatrix},$$

so $g = \begin{bmatrix} 400 \\ 300 \end{bmatrix} + \begin{bmatrix} 80 \\ 0 \end{bmatrix} = \begin{bmatrix} 480 \\ 300 \end{bmatrix}$. The ground speed is: $\|g\| = \sqrt{(480)^2 + (300)^2} \approx \boxed{566 \text{ mph}}$

The angle α the plane's ground motion makes with respect to East is given by $\tan(\alpha) = \frac{300}{480}$,

i.e. $\alpha = \arctan\left(\frac{300}{480}\right) \approx .5586$ radians $\approx \boxed{32 \text{ degrees}}$

---

3 Displacement vectors in the plane are

$$v = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \text{ and } w = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

A normal to the plane is

$$v \times w = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \vec{i} \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = \vec{i} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

(We expanded the determinant by minors along the first column.)

The equation can be:

$$\vec{n}_0 \circ (x - x_0, y - y_0, z - z_0) = 0$$

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \circ \begin{bmatrix} x-1 \\ y-0 \\ z-0 \end{bmatrix} = 0 \quad \therefore x - 1 = 0 \Rightarrow \boxed{x = 1}.$$

---

4 The component of u in the direction of v is:

$$u \circ \left(\frac{v}{\|v\|} \right). \quad \|v\| = \sqrt{1^2 + 1^2 + 0^2} = \sqrt{2}, \text{ so } \frac{1}{\|v\|} v = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix},$$

$$\text{so } u \circ \left(\frac{v}{\|v\|} \right) = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \circ \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix} = 2/\sqrt{2} + 1/\sqrt{2} = \boxed{3/\sqrt{2} = 3\sqrt{2}/2}.$$
