(1) (a) $\frac{\partial f}{\partial x} = -y^3 \sin(x,y)$, (b) $\frac{\partial f}{\partial y} = 2y\cos xy - xy^2\sin(xy)$

(c) $\frac{\partial^2 f}{\partial x^2} = -y^4\cos(x,y)$, (e) $\frac{\partial^2 f}{\partial x \partial y} = -3y^2\sin(xy) - xy^3\cos(x,y)$

(d) $\frac{\partial^2 f}{\partial y^2} = 2\cos(xy) - 2xy\sin(xy) - 2xy\sin(xy) - x^2y^2\cos(xy)$

$= 2\cos(xy) - 4xy\sin(xy) - x^2y^2\cos(xy)$

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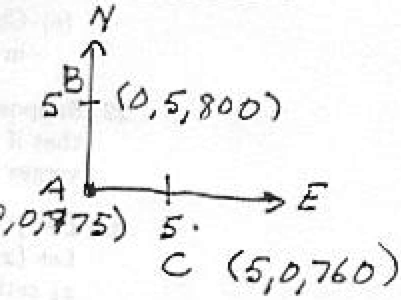
(2) ~~Let~~ Let $A(x,y)$ denote the altitude at point (x,y). Referring to the figure, we obtain



$$\nabla A(0,0) \approx \begin{bmatrix} (760-775)/5 \\ (800-775)/5 \end{bmatrix}$$

$$= \begin{bmatrix} -3 \\ 5 \end{bmatrix}$$, so an equation for the tangent plane is

$A = \cancel{800}\ 775 - 3X + 5Y$, or $\Delta A = -3\Delta X + 5\Delta Y$.

10 miles southwest corresponds to a direction and displacement of $\begin{bmatrix} \Delta X \\ \Delta Y \end{bmatrix} = 10 \begin{bmatrix} -1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$, so the altitude at that point would be approximately

$\cancel{800}\ 775 - 3(-10/\sqrt{2}) + 5(-10/\sqrt{2}) = 2(-10/\sqrt{2}) + \cancel{800}\ 775$

$\approx \cancel{800}\ 775 - 10\sqrt{2} \approx 761$ ft.

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(3) $\nabla f = \begin{bmatrix} \cancel{5(x}\ -3(x+1) + \frac{5}{\text{[illegible]}}y \\ 5(x+1) \end{bmatrix}$, $\nabla f(0,0) = \begin{bmatrix} -3 \\ 5 \end{bmatrix}$,

so an equation for the tangent plane is $z = 773.5 + \begin{bmatrix} -3 \\ 5 \end{bmatrix} \circ \begin{bmatrix} x-0 \\ y-0 \end{bmatrix}$

$= 773.5 - 3X + 5Y$

(The value of z is $773.5 - 10\sqrt{2}$, as in problem 2.)

4. The tangent plane is normal to $\nabla F(x,y,z)$, where $F(x,y,z) = xyz$.

$$\nabla F(x,y,z) = \begin{bmatrix} yz \\ xz \\ xy \end{bmatrix}, \text{ so } \nabla F(1,1,1) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

The tangent plane thus has an equation:

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \circ \begin{bmatrix} x-1 \\ y-1 \\ z-1 \end{bmatrix} = 0, \text{ which reduces to } \boxed{x+y+z=3},$$