Math. 302-03, Sixth exam answers, November 19, 2009

(1) A direction vector for the line is:

$$\vec{v} = \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix} - \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}.$$

Hence, a parametrization of the line is

$$\vec{r}(t) = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} + t \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \text{ or } \begin{aligned} x(t) &= -1+t \\ y(t) &= 2t \\ z(t) &= 1+3t. \end{aligned}$$

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(2) $$L = \int_0^{2\pi} \sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2}\, dt = \int_0^{2\pi} \sqrt{\cos^2(t) + \sin^2(t) + 1^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{2}\, dt = \boxed{2\sqrt{2}\pi}.$$

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(3) Its velocity vector is $\vec{r}\,'(t) = (-\pi \sin(\pi t), \pi \cos(\pi t), 2t)$, with $\vec{r}\,'(1) = (-\pi, 0, 2)$. Its speed is $\|\vec{r}\,'(1)\| = \sqrt{\pi^2+4}$. It is going in the direction $\frac{1}{\sqrt{\pi^2+4}}(-\pi, 0, 2)$.

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(4) Direction vectors for the plane are

$$\vec{v} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \text{ and } \vec{w} = \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}.$$

Parametric equations for the plane are thus

$$\vec{r}(s,t) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + s \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + t \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix},$$

or

$x(s,t) = 1+s+4t,$

$y(s,t) = 1+2s+5t,$

$z(s,t) = 1+3s+6t.$