Math. 302-03
Fall, 2014
R. B. Kearfott

Final Examination

Monday, December 8, 2014
2:00PM–4:30PM

Instructions: This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly and explained. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully. You may keep this exam sheet. Each entire problem is worth 14 points, and 2 points are free.

1. Design a rectangular carton of width w, length ℓ, and height h to hold 2000 cubic centimeters (i.e. 2 liters). The sides of the box cost \$0.05 per square centimeter and the bottom and top cost \$0.10 per square centimeter. Find the dimensions of the box that minimize the total cost of materials used. Show all work.

2. Evaluate the integral
$$\int_{y=0}^{1} \int_{x=\sqrt{y}}^{1} \sqrt{2+x^3} dx dy$$
by reversing the order of integration.

3. Evaluate
$$\int\int\int_{\mathcal{V}} \sqrt{x^2+y^2+z^2} dV,$$
where $\mathcal{V}$ is the top half of the interior of the sphere $x^2+y^2+z^2=9$.

4. Write down parametric equations for the circle of radius 3 centered at $(x,y)=(1,2)$, oriented counterclockwise.

5. A particle travels along the line $x=1+t$, $y=5+2t$, $z=-7+t$, where t is in seconds and x, y, and z are in meters.
   (a) When and where does the particle hit the plane $x+y+z=1$?
   (b) How fast is the particle going when it hits the plane? Give units.

6. Compute the flux of the vector field
$$\vec{F}(x,y,z) = (x, y, \cos(x^2+5y^2))$$
out of the closed cylinder of radius 2 with axis the z-axis and extending between $z=0$ and $z=3$.

7. Write down the circulation of the vector field
$$\vec{F}(x,y,z) = (yz, xz, xy)$$
about the closed curve
$$x=\cos(t), \quad y=\sin(t), \quad z=\sin(2t), \quad 0 \le t \le 2\pi$$
Be sure so show all computations leading to your answer.