Second Examination
Friday, October 3, 2014

Instructions: This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly and explained. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully. You may keep this exam sheet. Each entire problem is worth 25 points.

1. Compute the gradient vector of the following functions.

   (a) $f(x, y) = ye^{x+y}$ (b) $f(x, y) = x^2 y$ (c) $f(x, y) = \arctan(x^2 - y^2)$.

2. The value of a certain quantity at point (x, y) is given approximately by:
$$f(x, y) = x^2 + 2y^2.$$
   What is the rate of increase of f at $(1, 1)$ in the direction $(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$?

3. Write down an equation for the tangent plane to
$$f(x, y) = x^2 - y^2$$
   at the point $(2, 1)$.

4. Suppose a person's position at time t is given by the coordinates
$$(x(t), y(t)) = (t, t^2),$$
   while the altitude at point (x, y) is given by
$$A(x, y) = 10,000 - x^2 - y^2.$$
   What is the rate of change of altitude of the person at time $t = 1$?