Math. 302-03
Fall, 2014
R. B. Kearfott

Sixth Examination

Monday, November 10, 2014

Instructions: This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly and explained. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully. You may keep this exam sheet. Each entire problem is worth 33 points, and 1 point is free.

1. Find the line integral
$$\int_{\mathcal{C}} (x\vec{i} + y\vec{j}) \cdot d\vec{r},$$
where $\mathcal{C}$ is the top half of the circle of radius 2 centered at the origin and oriented counterclockwise. Besides showing your computations, also sketch a graph of the oriented path $\mathcal{C}$.

2. If $\vec{F}(x,y) = 4x(x^2+y)\vec{i} + 2(x^2+y)\vec{j}$, find $\displaystyle\int_{\mathcal{C}} \vec{F} \cdot d\vec{r}$, where $\mathcal{C}$ is the composite curve consisting of:
   (a) the line from $(0,0)$ to $(1,0)$,
   (b) followed by the arc of the circle of radius 1 centered at the origin oriented counterclockwise from $(1,0)$ to $(0,1)$,
   (c) followed by the line segment from $(0,1)$ to $(2,4)$,
   (d) followed by the curve on the parabola $y = x^2$ proceeding from $(2,4)$ to $(1,1)$.

   (*Hint: Can you take a short cut? If so, explain why it is allowed.*)

3. If
$$\vec{F}(x,y) = (-\cos(y) + y^3/3 - xy)\vec{i} + (x\sin(y) + xy^2 - y)\vec{j},$$
find $\displaystyle\int_{\mathcal{C}} \vec{F} \cdot d\vec{r}$, where $\mathcal{C}$ is the closed curve consisting of the line segment from $(0,0)$ to $(1,0)$, followed by the line segment from $(1,0)$ to $(1,1)$, followed by the line segment from $(1,1)$ to $(0,1)$, followed by the line segment from $(0,1)$ back to $(0,0)$.
(*Hint: Can you express the line integral as another kind of integral that is easier to compute?)*