(1) (a) $\frac{\partial f}{\partial x} = ye^{x+y}$, $\frac{\partial f}{\partial y} = e^{x+y} + ye^{x+y} = (1+y)e^{x+y}$;

so $\boxed{\nabla f = (ye^{x+y}, (1+y)e^{x+y})}$

(b) $\boxed{\nabla f = (2xy, x^2)}$

(c) $\boxed{\nabla f = \left(\frac{2x}{(x^2-y^2)^2+1}, \frac{-2y}{(x^2-y^2)^2+1} \right)}$

(2) $\nabla f = (2x, 4y)$; $\nabla f(1,1) = (2,4)$

$\nabla f(1,1) \circ \left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right) = 2\left(\frac{1}{\sqrt{2}}\right) - 4\left(\frac{1}{\sqrt{2}}\right) = \frac{-2}{\sqrt{2}} = \boxed{-\sqrt{2}}$.

(3) $\nabla f = (2x, -2y)$; $\nabla f(2,1) = (4,-2)$

$f(2,1) = 3$, so an equation for the tangent plane is: $z = 3 + (4,2) \circ (x-2, y-1)$

i.e $\boxed{z = 3 + 4(x-2) - 2(y-1)}$.

(4) We use the multivariate chain rule:

$$\frac{dA(x(t), y(t))}{dt} = \frac{\partial A}{\partial x}\frac{dx}{dt} + \frac{\partial A}{\partial y}\frac{dy}{dt}$$

$= -2x(1) - 2y(2t)$

$= -2t - 4t^3$, so

$$\left.\frac{dA}{dt}\right|_{t=1} = \boxed{-6}$$