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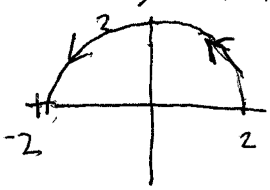
① We may parametrize the circle by

$x = 2\cos(t),\ y = 2\sin(t),\quad 0 \le t \le \pi.$

Then $d\vec{r} = (-2\sin t, 2\cos t)\,dt$, and

$$\int_{\mathcal{C}} \vec{F}\circ d\vec{r} = 4\int_{t=0}^{\pi} (\cos(t), \sin(t))\circ(-\sin(t), \cos(t))\,dt$$

$$= 4\int_{t=0}^{\pi} 0\,dt = \boxed{0}$$



② The line integral will become easy to compute if $\vec{F}$ has a potential function, so let's try to find one. Taking $\int F_1\,dx$ gives:

$f(x,y) = (x^2+y)^2 + g(y),$

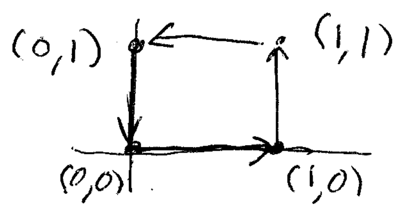
so $f_y(x,y) = 2(x^2+y) + g'(y) = 2(x^2+y).$

Thus, $g'(y) = 0$, so $f(x,y) = (x^2+y)^2 + C$.

Therefore, by the fundamental theorem of line integrals, we have:

$$\int_{\mathcal{C}} \vec{F}\circ d\vec{r} = f(1,1) - f(0,0) = (1+1)^2 - 0 = \boxed{4}$$

③ We use Green's theorem:



$$\int_{\mathcal{C}} \vec{F}\circ d\vec{r} = \int_{x=0}^{1}\int_{y=0}^{1}\left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)dy\,dx$$

$$= \int_{x=0}^{1}\int_{y=0}^{1} (\sin(y)+y^2) - (-x)\,dy\,dx = \int_{x=0}^{1}\int_{y=0}^{1} x\,dy\,dx$$

$$= \int_{x=0}^{1} x\,dx = \boxed{\frac{1}{2}}$$