Math 302-03, Fall 2014. Third exam answers, page 1

(1) $\frac{\partial f}{\partial x} = 2x - y = 0$; $\frac{\partial f}{\partial y} = 2y - x = 0 \Rightarrow y = 2x,\ 4x - x = 0$

$\therefore x = 0, y = 0$ is the only critical point.

$\frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 f}{\partial y^2} = 2 > 0$. $\frac{\partial^2 f}{\partial x \partial y} = -1$. $D = (2)(2) - (-1)^2 = 3 > 0$.

Therefore, $(0,0)$ corresponds to a local minimum.

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(2) We have the distance squared:

$$f(x,y) = (x^2) + (y^2) + (1-x-y)^2$$

$$\frac{\partial f}{\partial x} = 2x - 2(1-x-y) = 0 = 4x + 2y - 2$$

$$\frac{\partial f}{\partial y} = 2y - 2(1-x-y) = 0 = 2x + 4y - 2$$

i.e. $\left\{\begin{matrix} 2x + y = 1 \\ x + 2y = 1 \end{matrix}\right\} \Leftrightarrow \left\{\begin{matrix} x + 2y = 1 \\ 2x + y = 1 \end{matrix}\right\} \Leftrightarrow \left\{\begin{matrix} x + 2y = 1 \\ -3y = -1 \end{matrix}\right\}$

$\boxed{y = \frac{1}{3}, x = \frac{1}{3}, z = \frac{1}{3}}$.

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(3) $\frac{\partial f}{\partial x} = 2x = \lambda \frac{\partial g}{\partial x} = \lambda$

$\frac{\partial f}{\partial y} = 2y = \lambda \frac{\partial g}{\partial y} = \lambda$

$\frac{\partial f}{\partial z} = 2z = \lambda \frac{\partial g}{\partial z} = \lambda$

$2x = 2y = 2z \Rightarrow$

$x = y = z$.

Since $x + y + z = 1$, this gives

$\boxed{x = y = z = \frac{1}{3}}$
