

Second Examination

Thursday, September 29, 2016

Instructions: This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly and explained. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully. You may keep this exam sheet. Each entire problem is worth 14 points, and 2 points are free.

1. Sketch the graph in the plane of $\mathbf{r}(t) = t^2\mathbf{i} + t\mathbf{j}$, $-1 \leq t \leq 1$, and indicate the direction of increasing t with an arrow on the graph.
2. Find parametric equations for the line tangent to the graph of $\mathbf{r}(t) = \langle t^2, t, \sin(\pi t) \rangle$ at $t = 1$.
3. Sketch the contours (level curves) $f(x, y) = c$ of the function $f(x, y) = x^2 - y^2$ for $c = -1$, $c = 0$, and $c = 1$.
4. Determine if the limit exists, and if it does, find it:

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + 6xy + 9y^2}{x + 3y}.$$

5. Compute the following partial derivatives:
 - (a) $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$, where $f(x, y) = x(x^2 + y)$.
 - (b) $g_x(1, \pi)$, where $g(x, y) = x \cos(xy)$.
6. If $f(1, 1) = 1$, $f_x(1, 1) = -1$, and $f_y(1, 1) = 2$, approximate $f(1.2, 0.8)$.
7. Use the chain rule to compute $g'(1)$, where $g(t) = f(x(t), y(t))$, $f(x, y) = (x - y)^2$, $x(t) = (t - 2)^{50}$, $y(t) = (t^2 + t + 1)$.