

Fifth Examination

Tuesday, November 28, 2016

Instructions: This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly and explained. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully. You may keep this exam sheet. Each entire problem is worth 33 points, and 1 point is free.

1. Use Stokes' Theorem to compute the work the force field

$$\vec{F}(x, y, z) = \langle -y, x, z \rangle$$

does on a particle traveling along the boundary of the unit square in the x - y plane, where the particle goes from

$P_0(0, 0, 0)$ to $P_1(1, 0, 0)$ to $P_2(1, 1, 0)$ to $P_3(0, 1, 0)$ to $P_0(0, 0, 0)$.

2. Use the divergence theorem to compute the flux of the vector field in Problem 1 out of the unit sphere $x^2 + y^2 + z^2 \leq 1$.

3. Compute

$$\int \int_{\sigma} f(x, y, z) dS,$$

where

$$f(x, y, z) = z$$

and σ is defined parametrically by

$$\sigma : \quad x = u, \quad y = u, \quad z = u + v^2 \quad \text{with } 0 \leq u \leq 1, \quad 0 \leq v \leq 2.$$