(1) $(x^2+4x+4)-4+(y^2-8y+16)-16+(z^2+6z+9)-9=7.$

$(x+2)^2-4+(y-4)^2-16+(z+3)^2-9=7$

$(x+2)^2+(y-4)^2+(z+3)^2=36=6^2$

this corresponds to a sphere of radius 6 centered at $(-2,4,-3)$.

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(2) a non-unit vector in that direction is:

$\vec{v}=\langle -1,-2,-3\rangle-\langle 1,2,-1\rangle=\langle -2,-4,-2\rangle.$

$\|\vec{v}\|=\sqrt{(-2)^2+(-4)^2+(-2)^2}=\sqrt{4+16+4}=\sqrt{24}=2\sqrt{6}.$

a corresponding unit vector is thus:

$\vec{u}=\frac{1}{2\sqrt{6}}\langle -2,-4,-2\rangle=\boxed{\left\langle \frac{-1}{\sqrt{6}},\frac{-2}{\sqrt{6}},\frac{-1}{\sqrt{6}}\right\rangle}$

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(3) $\vec{c}=\frac{1}{\|\vec{b}\|}\vec{b}=\frac{1}{\sqrt{2}}\langle -1,0,1\rangle=\left\langle \frac{-1}{\sqrt{2}},0,\frac{1}{\sqrt{2}}\right\rangle=\vec{c}$

$\vec{v}=(\vec{a}\circ\vec{c})\vec{c}=\left(\langle 1,2,3\rangle\circ\left\langle\frac{-1}{\sqrt{2}},0,\frac{1}{\sqrt{2}}\right\rangle\right)\left\langle\frac{-1}{\sqrt{2}},0,\frac{1}{\sqrt{2}}\right\rangle$

$=\sqrt{2}\left\langle\frac{-1}{\sqrt{2}},0,\frac{1}{\sqrt{2}}\right\rangle=\boxed{\langle -1,0,1\rangle=\vec{v}}$

$\vec{w}=\vec{a}-\vec{v}=\langle 1,2,3\rangle-\langle -1,0,1\rangle=\boxed{\langle 2,2,2\rangle=\vec{w}}$

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(4) $\vec{r}(t)=\langle 1,2,3\rangle+t(\langle 4,5,6\rangle-\langle 1,2,3\rangle)$, i.e.

$\boxed{\vec{r}(t)=\langle 1,2,3\rangle+t\langle 3,3,3\rangle}.$

alternatively, $\vec{r}(t)=t\langle 4,5,6\rangle+(1-t)\langle 1,2,3\rangle$

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(5) Vectors parallel to the plane are $\vec{v}=\langle 1,0,1\rangle-\langle 0,0,0\rangle=\langle 1,0,1\rangle$ and $\vec{w}=\langle 1,2,0\rangle-\langle 0,0,0\rangle=\langle 1,2,0\rangle$. a normal to the plane is thus:

$\vec{n}=\begin{vmatrix}\vec{i}&\vec{j}&\vec{k}\\1&0&1\\1&2&0\end{vmatrix}=\vec{i}\begin{vmatrix}0&1\\2&0\end{vmatrix}-\vec{j}\begin{vmatrix}1&1\\1&0\end{vmatrix}+\vec{k}\begin{vmatrix}1&0\\1&2\end{vmatrix}$

$=-2\vec{i}+\vec{j}+2\vec{k}=\langle -2,1,2\rangle$, so an equation for the plane is $\langle -2,1,2\rangle\circ(x-0,y-0,z-0)=0$,

that is, $\boxed{-2x+y+2z=0}$.

(6) $\rho^2 = x^2+y^2+z^2 = \frac{1}{2}+\frac{1}{2}+1 = 2$, so $\boxed{\rho = \sqrt{2}}$.

$r = \sqrt{x^2+y^2}$, and $\tan\varphi = \frac{r}{z} = \frac{1}{-1} = -1$,

so $\varphi = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$. $\tan(\theta) = \frac{y}{x} = 1 \Rightarrow \theta = \frac{\pi}{4}$.

The spherical coordinates are thus:

$$\boxed{(\rho, \theta, \varphi) = \left(\sqrt{2}, \frac{\pi}{4}, \frac{3\pi}{4}\right)}$$