① $S(u,v) = (u, u, u+v^2)$, so $\nabla_u(S) = (1,1,1)$ and $\nabla_v(S) = (0,0,2v)$. Thus,

$$\nabla_u(S) \times \nabla v(S) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 0 & 0 & 2v \end{vmatrix} = 2v(\vec{i} - \vec{j}),$$

so $\|\nabla_u S \times \nabla_v S\| = 2v\sqrt{2}$. Thus, the surface area is

$$\int_{u=0}^{1}\int_{v=0}^{2} 2\sqrt{2}\, v\, dv\, du = \int_{u=0}^{1} \sqrt{2}\, v^2 \Big|_{v=0}^{2} du = \boxed{4\sqrt{2}}$$

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② $z = 1-y$, $0 \le x \le 1$, $0 \le y \le 1$ $0 \le z \le 1-y$, so the integral is

$$\int_{x=0}^{1} x \int_{y=0}^{1}\int_{z=0}^{1-y} dz\, dy\, dx = \int_{x=0}^{1} x \int_{y=0}^{1} (1-y)\, dy\, dx$$

$$= \int_{x=0}^{1} x\left[y - \frac{y^2}{2}\right]\Big|_{y=0}^{1} dx = \int_{x=0}^{1} \frac{1}{2} x\, dx = \frac{1}{2}\left[\frac{1}{2}x^2\right]\Big|_{x=0}^{1} = \boxed{\frac{1}{4}}.$$

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③ $$\iiint_{\mathcal{Q}} e^{-(x^2+y^2+z^2)^{3/2}} dV = \int_{\rho=0}^{1}\int_{\theta=0}^{2\pi}\int_{\varphi=0}^{\pi/2} e^{-\rho^3}\rho^2 \sin\varphi\, d\varphi\, d\theta\, d\rho$$

$$= \left(\int_{\rho=0}^{1} e^{-\rho^3}\rho^2\, d\rho\right)\left(\int_{\theta=0}^{2\pi} d\theta\right)\left(\int_{\varphi=0}^{\pi/2} \sin\varphi\, d\varphi\right) = \frac{1}{3}\left(\int_{u=0}^{1} e^{-u}\, du\right)(2\pi)(1)$$

$$= \frac{1}{3}(1-e^{-1})(2\pi) = \boxed{\frac{2}{3}\pi(1-e^{-1})}$$

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④ $d\vec{r} = (1,t,t^2)\,dt$, so $\displaystyle\int_{\mathcal{C}} \vec{F} \circ d\vec{r} = \int_{t=1}^{2} \left(1, \frac{1}{t}, \frac{1}{t^2}\right) \circ (1,t,t^2)\, dt$

$$= \int_{t=1}^{2} 3\, dt = \boxed{3}$$

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5 (a) $\frac{\partial(y+2x)}{\partial y}=1$, $\frac{\partial(x+1)}{\partial x}=1=\frac{\partial(y+2x)}{\partial y}$. Thus, $\vec{F}$ is conservative.

a potential function is $\int y+2x\,dx = xy+x^2+g(y)$,

so we have $\frac{\partial}{\partial y}(xy+x^2+g(y)) = x+g'(y)=x+1$,

so $g'(y)=1$. This gives $g(y)=y+C$, so potential functions are of the form $\boxed{\varphi(x,y)=xy+x^2+y+C}$

(b) Since $\mathcal{C}$ is a closed curve and $\vec{F}$ is conservative,

$$\boxed{\int_{\mathcal{C}} \vec{F}\circ d\vec{r} = 0}$$

(c) Since $\vec{F}$ is conservative,

$$\int_{\mathcal{C}} \vec{F}\circ d\vec{r} = \varphi(\vec{r}(1))-\varphi(\vec{r}(0)) = \varphi(1,0)-\varphi(0,0)$$
$$= (1)-0 = \boxed{1}$$