Math. 302-05, Fall, 2018, Third exam answers, page 1

(1) $\nabla f = \langle 2x+y, x+2y, -1\rangle$, so $\nabla f(1,2,0) = \langle 4,5,-1\rangle$,
and $D_{\vec{u}} f(1,2,0) = \langle 4,5,-1\rangle \cdot \langle +1/\sqrt{3}, -1/\sqrt{3}, 1/\sqrt{3}\rangle$
$= 4/\sqrt{3} - 5/\sqrt{3} - 1/\sqrt{3} = \boxed{-2/\sqrt{3}}$

(2) $\nabla f = \langle \sin xy + xy\cos xy,\ x^2\cos(xy)\rangle$
so $\nabla f(1,\pi) = \langle -\pi, -1\rangle$. $f(1,\pi)=0$, so an equation for the tangent plane is: $z-0 = -\pi(x-1) - 1(y-\pi)$.
In standard form, this is: $\boxed{\pi x + y + z = 2\pi}$.
Thus, two normal vectors are: $\boxed{\langle \pi,1,1\rangle \text{ and } \langle -\pi,-1,-1\rangle}$.

(3) $\nabla f = \langle x-y, -x+1\rangle = \langle 0,0\rangle$ where $\left\{\begin{matrix} x-y=1 \\ x=1\end{matrix}\right\}$
Thus, the only critical point is at $\boxed{(1,0)}$. To classify it, we have:
$f_{xx}=1$, $f_{yy}=0$, $f_{xy}=-1$. Thus, $D = f_{xx}f_{yy} - (f_{xy})^2 = -1 < 0$,
so the critical point corresponds to a $\boxed{\text{saddle point}}$.

(4) $\nabla f = \langle 2x, -2y\rangle$; $\nabla g = \langle 2x, 2y\rangle$
$\nabla f = \lambda\nabla g \Rightarrow 2x = \lambda(2x)$, $-2y = \lambda(2y)$
Thus, $x=0$ or $\lambda=1$, If $x=0$, the second equation gives $y=0$ or $\lambda=-1$. $x=0, y=0$ cannot occur, because it doesn't satisfy $g=9$. Thus, we have $(0,\pm3)$ as critical points. Analyzing the second equation, $y=0$ or $\lambda=-1$. If $y=0$, the constraint gives $x=\pm3$, while $\lambda=-1$ yields $x=0$. Thus, we have 4 critical points.

(x,y)	$f(x,y)$
$(0,-3)$	-9
$(0,3)$	-9
$(-3,0)$	9
$(3,0)$	9

Thus, the min is -9, occurring at $(0,-3)$ and $(0,3)$, and the max is 9, occurring at $(3,0)$ and $(-3,0)$

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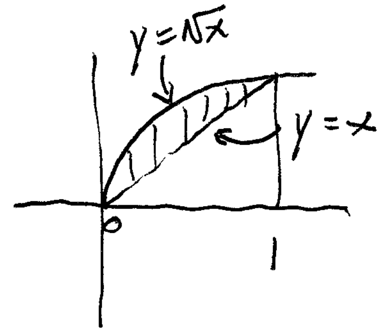
$$\iint_R xy\,dA = \int_{x=0}^{1}\left\{\int_{y=x}^{\sqrt{x}} x\,y\,dy\right\}dx$$



$$= \int_{x=0}^{1} \frac{x}{2}\left[y^2\right]\Big|_{y=x}^{\sqrt{x}} dx = \frac{1}{2}\int_{x=0}^{1} x\left[x-x^2\right]dx$$

$$= \frac{1}{2}\int_{0}^{1} x^2-x^3\,dx = \frac{1}{2}\left[\frac{x^3}{3}-\frac{x^4}{4}\right]\Big|_{x=0}^{1} = \frac{1}{2}\left[\frac{1}{3}-\frac{1}{4}\right] = \frac{1}{2}\left[\frac{1}{12}\right] = \boxed{\frac{1}{24}}$$

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$$\iint_R \sqrt{9-x^2-y^2}\,dA = \int_{r=0}^{3}\int_{\theta=0}^{\pi/2}\sqrt{9-r^2}\,r\,d\theta\,dr$$

$$= \frac{\pi}{2}\int_{r=0}^{3}\sqrt{9-r^2}\,r\,dr$$

$$\boxed{\begin{array}{l} u = 9-r^2;\ du = -2r\,dr \\ \quad -\frac{du}{2} = r\,dr \end{array}}$$

$$= \frac{\pi}{2}\int_{u=9}^{0}\sqrt{u}\left(-\frac{du}{2}\right) = \frac{\pi}{4}\int_{u=0}^{9} u^{1/2}\,du = \frac{\pi}{4}\left(\frac{2}{3}u^{3/2}\right)\Big|_{u=0}^{9} = \frac{\pi}{6}\left[27\right] = \boxed{\frac{9}{2}\pi}$$