First Examination
Monday, September 18, 2017

Instructions: This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly and explained. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully. You may keep this exam sheet. Each part of each problem is worth 10 points.

1. Write down an equation for the sphere with center $(1, -1, 3)$ and radius 2.

2. Describe as precisely as possible the surface given by the equation

$$x^2 + y^2 + z^2 + 6x + 8y = 0.$$

3. Find

   (a) the projection of $\langle 3, 4, 6 \rangle$ onto $\langle 1, 1, 0 \rangle$, and

   (b) find the length of that projection.

4. Consider the points $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$.

   (a) Use the cross product to find the area of the triangle with these points as vertices.

   (b) Find a unit normal vector to the plane in which this triangle lies.

5. Show that the following lines intersect, and find their point of intersection.

$$
\begin{array}{cccccc}
x & = & t, & y & = & t, & z & = & t, \\
x & = & 2u - 1, & y & = & -u, & z & = & -u.
\end{array}
$$

6. Determine an equation for the plane perpendicular to the line through $(0, 0, 0)$ and $(1, 1, 1)$ and containing the point $(2, 3, 4)$.

7. Consider the spherical coordinate equation $\rho \sin(\varphi) = 1$.

   (a) Write down a corresponding equation in rectangular coordinates.

   (b) Precisely describe the graph of this equation.