Math. 302-04
Fall, 2017
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Second Examination
Tuesday, October 10, 2017

Instructions: This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly and explained. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully. You may keep this exam sheet. Each part of each problem is worth 12 points, and 4 points are free.

1. Consider
$$\boldsymbol{r}(t) = \left\langle \sqrt{t^2-1}, \log(2t-1), \sin(t) \right\rangle .$$
   (a) Find the domain of $\boldsymbol{r}$, that is, specify the set where $\boldsymbol{r}$ is defined.
   (b) Compute $\boldsymbol{r}'(t)$.
2. Find an arc-length parametrization of the line given by $x = 1 + 3t$, $y = 2 - 4t$.
3. Find $f(u, v)$ if $f(x, y) = x^2 + y^2$, $u = r\cos(\theta)$, and $v = r\sin(\theta)$. Simplify the expression in r and θ as much as possible.
4. If $f(x, y, z) = x\ln(y^2 \sin(z))$, find $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, and $\frac{\partial f}{\partial z}$.
5. If it is known $f(1, 2) = 3$, $f_x(1, 2) = -2$, and $f_y(1, 2) = 4$, use the differential to compute an approximation to $f(0.95, 2.08)$.
6. The temperature at point (x, y, z) in space is given by
$$T(x, y, z) = x + xy + xyz.$$
   A particle's position in space at time t is given as
$$\boldsymbol{r}(t) = \langle x(t), y(t), z(t)\rangle = \langle \sin(\pi t), \cos(\pi t), t\rangle .$$
   Use the multivariate chain rule to compute the particle's rate of change of temperature at time $t = 1$. That is, compute $\left.\frac{dT}{dt}\right|_{t=1}$.
7. Derive an equation for the tangent plane to $f(x, y) = y^2 - x^2$ at $(x, y) = (1, 2)$.