Math. 302-04
Fall, 2017
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Fourth Examination
Tuesday, November 28, 2017

Instructions: This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly and explained. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully. You may keep this exam sheet. Each problem is worth 33 points, and 1 point is free.

1. Use the divergence theorem to compute the surface integral
$$\iint_{\mathcal{S}} \vec{F}\cdot\vec{n}\, dS,$$
where $\vec{F} = \langle \frac{1}{3}x^3, \frac{1}{3}y^3, \frac{1}{3}z^3\rangle$ and $\mathcal{S}$ is the unit sphere
$$\left\{(x,y,z) \mid x^2+y^2+z^2=1\right\}.$$

2. Use Green's theorem to compute
$$\oint_{\mathcal{C}} \vec{F}\cdot d\vec{r},$$
where $\vec{F} =< x, x >$ and $\mathcal{C}$ is the square oriented by going from $(0,0)$ to $(1,0)$ to $(1,1)$ to $(0,1)$ then back to $(0,0)$.

3. Use Stokes' theorem to compute the circulation of the vector field $\vec{F} = \langle x, y, y\rangle$ around the rectangle traversed from $(x,y,z) = (1,0,0)$ to $(1,1,0)$ to $(1,1,1)$ to $(1,0,1)$ and back to $(1,0,0)$.