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① (a) The domain is $\{t \mid -1 \le t \le 1\} \cap \{t \mid 2t-1 > 0\} \cap \mathbb{R}$

$= [-1,1] \cap (1/2, \infty) \cap (-\infty, \infty) = \boxed{(1/2, 1]}$.

(b) $\vec{r}\,'(t) = \left\langle \frac{2t}{2\sqrt{t^2-1}}, \frac{1}{2t-1}(2), \cos(t) \right\rangle = \left\langle \frac{t}{\sqrt{t^2-1}}, \frac{1}{t-1/2}, \cos(t) \right\rangle$

② $\vec{r}(t) = \langle 1,2\rangle + \langle 3,-4\rangle t$. $\vec{r}\,'(t) = \langle 3,-4\rangle$, $\|\vec{r}\,'(t)\| = 5 = \sqrt{3^2+4^2}$

Thus $s(t) = \int_0^t \|\vec{r}\,'(u)\|\,du = \int_0^t 5\,du = 5t$, or $t = \frac{s}{5}$.

An arc length parametrization is thus

$\vec{r}(s) = \langle 1,2\rangle + \langle 3,-4\rangle\left(\frac{s}{5}\right)$, or $\boxed{x(s) = 1 + \frac{3}{5}s,\ y(s) = 2 - \frac{4}{5}s}$

③ $f(u,v) = (r\cos(\theta))^2 + (r\sin(\theta))^2 = r^2\cos^2(\theta) + r^2\sin^2(\theta)$

$= r^2(\cos^2\theta + \sin^2\theta) = \boxed{r^2}$.

④ $\frac{\partial f}{\partial x} = \ln(y^2 \sin z)$; $\cancel{\frac{\partial f}{\partial y} = \frac{x}{y^2\sin(z)}}$ $\cancel{\frac{2x}{y\sin(z)}}$ $\cancel{2y\sin(z)}$

$\cancel{\frac{\partial f}{\partial z} = \frac{x}{y^2\sin z}\cos(z)}$

$\frac{\partial f}{\partial y} = \frac{x}{y^2\sin z} \cdot 2y\sin(z) = \frac{2x}{y}$.

$\frac{\partial f}{\partial z} = \frac{x}{y^2\sin(z)} \cdot y^2\cos(z) = x\cot(z)$

⑤ $f(0.95, 2.08) \approx 3 - 2(-0.05) + 4(0.08)$

$= 3 + 0.1 + 0.32 = 3.42$.

⑥ $T_x = 1 + y + yz$; $T_y = x + xz$; $T_z = xy$.

$\vec{r}(1) = \langle \sin(\pi), \cos\pi, 1\rangle = \langle 0,-1,1\rangle$,

and $\nabla T(0,-1,1) = \langle 1 - 1 + (-1)(1), 0 + 0(1), (0)(-1)\rangle = \langle -1, 0, 0\rangle$.

$\frac{dT}{dt} = \nabla T(x(t), y(t), z(t)) \circ \vec{r}\,'(t)$.

$\vec{r}\,'(t) = \langle \pi\cos(\pi t), -\pi\sin(\pi t), 1\rangle$

so $\vec{r}\,'(1) = \langle -\pi, 0, 1\rangle$, so $\left.\frac{dT}{dt}\right|_{t=1} = \langle -1,0,0\rangle \circ \langle -\pi, 0, 1\rangle = \boxed{\pi}$

(7) $f_x = 2x$; $f_y = 2y$. $f(1,2) = 2^2 - 1^2 = 3$, $f_x(1,2) = -2$, $f_y(1,2) = 4$.

Thus, an equation for the tangent plane is:

~~$-2(x-1)+4($~~ $2(x-1) - 4(y-2) + (z-3) = 0$,

that is, $2x - 2 - 4y + 8 + z - 3 = 0$, or

$\boxed{x - 4y + z = -3}$