1. ~~Normal vectors to~~ Vectors in the plane are $\langle 1,1,1\rangle - \langle 0,2,0\rangle = \langle 1,-1,1\rangle$ and $\langle 1,1,1\rangle - \langle 0,0,3\rangle = \langle 1,1,-2\rangle$. A normal vector is thus

$$\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 1 \\ 1 & 1 & -2 \end{vmatrix} = \vec{i}\begin{vmatrix} -1 & 1 \\ 1 & -2 \end{vmatrix} - \vec{j}\begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} + \vec{k}\begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}$$
$$= \vec{i} + 3\vec{j} + 2\vec{k}$$

An equation for the plane is thus:

$\langle 1,3,2\rangle \circ (\langle x,y,z\rangle - \langle 1,1,1\rangle) = 0$, or $\boxed{x+3y+2z=6}$.

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2. (a) $\vec{w} = \dfrac{\vec{b}\circ\vec{v}}{\|\vec{b}\|^2}\vec{b}$. $\vec{b}\circ\vec{v} = -1+0+2=1$, $\|\vec{b}\|^2 = \vec{b}\circ\vec{b} = 1+4=5$,

so $\vec{w} = \frac{1}{5}\langle -1,0,2\rangle = \boxed{\langle \frac{-1}{5}, 0, \frac{2}{5}\rangle}$

(b) Let $\vec{w}^{\perp}$ be the component of $\vec{v}$ orthogonal to $\vec{b}$.

Then $\vec{w}^{\perp} = \langle 1,1,1\rangle - \langle \frac{-1}{5}, 0, \frac{2}{5}\rangle = \boxed{\langle \frac{6}{5}, 1, \frac{3}{5}\rangle}$

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3. The volume is $|\vec{u}\circ(\vec{v}\times\vec{w})| = \left|\langle 1,0,0\rangle \circ \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{vmatrix}\right|$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{vmatrix} = 1\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = \boxed{1}$$

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4. A point on both lines, and hence in the plane, is where $\{t=-u,\ t=2u-1\} \Rightarrow -u = 2u-1 \Rightarrow u=\frac{1}{3}, t=\frac{-1}{3}$. That point is $\langle \frac{-1}{3}, \frac{-1}{3}, \frac{-1}{3}\rangle$. Directions in the plane are $\langle 1,1,1\rangle$ and $\langle 2,-1,-1\rangle$, so a normal is

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 2 & -1 & -1 \end{vmatrix} = \vec{i}\begin{vmatrix} 1 & 1 \\ -1 & -1 \end{vmatrix} - \vec{j}\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} + \vec{k}\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix}$$
$$= 0\vec{i} + 3\vec{j} - 3\vec{k} = \langle 0,+3,-3\rangle$$

An equation for the plane is thus:

$\langle 0,+3,-3\rangle \circ \langle x+\frac{1}{3}, y+\frac{1}{3}, z+\frac{1}{3}\rangle = 0$,

that is, $\boxed{\cancel{y+z=-\frac{2}{3}}}$ $\boxed{y-z=0}$

(5) The intersection is where

$$2(t-1)-(-t)+(2t+1)=3$$

$$2t-2+t+2t+1=3 \Leftrightarrow 5t=4 \Rightarrow \boxed{t=\tfrac{4}{5}}$$

The point of intersection is thus:

$$\left\langle \tfrac{4}{5}-1, -\tfrac{4}{5}, 2\left(\tfrac{4}{5}\right)+1 \right\rangle = \boxed{\left\langle -\tfrac{1}{5}, -\tfrac{4}{5}, \tfrac{13}{5} \right\rangle}$$

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(6) (a) $\rho^2 = \rho\sin\varphi\cos\theta + \rho\sin\varphi\sin\theta + \rho\cos\varphi$

or $\rho = \sin\varphi(\cos\theta + \sin\theta) + \cos\varphi$

(b) $x^2 - x + y^2 - y + z^2 - z = 0$

$$\left(x-\tfrac{1}{2}\right)^2 + \left(y-\tfrac{1}{2}\right)^2 + \left(z-\tfrac{1}{2}\right)^2 = \tfrac{3}{4}.$$

The graph is a sphere of radius $\frac{\sqrt{3}}{2}$ centered at $\left(\tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}\right)$
