1. $\vec{r}(t)=\langle t,t^2\rangle$, $0\le t\le 1$. Thus, the work is

$$\int_{\mathcal{C}} \langle xy, x^2\rangle \circ d\vec{r} = \int_{t=0}^{1} \langle (t)(t^2), t^2\rangle \circ \langle 1, 2t\rangle\, dt$$

$$= \int_{t=0}^{1} t^3+2t^3\, dt = 3\int_{t=0}^{1} t^3\, dt = \boxed{\frac{3}{4}}.$$

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2. $\dfrac{\partial(2xy)}{\partial y} = 2x = \dfrac{\partial x^2}{\partial x}$, so the vector field is conservative.

a potential function satisfies

$$\varphi(x,y)=x^2y+g(y),\quad \frac{\partial\varphi}{\partial y}=x^2+g'(y)=x^2 \Rightarrow g'(y)=0$$

$$\Rightarrow \varphi(x,y)=x^2y+C.$$

Thus

$$\int_{(0,0)}^{(1,1)} \vec{F}\circ d\vec{r} = \varphi(1,1)-\varphi(0,0) = 1^2(1)-(0)^2(0) = \boxed{1}$$

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3. We use Green's theorem:

$$\oint_{\mathcal{C}} \vec{F}\circ d\vec{r} = \iint_R \frac{\partial g}{\partial x}-\frac{\partial f}{\partial y}\, dA = \iint_R 1-(-1)\, dA$$

$$= 2\,[\text{Area of unit circle}] = 2(\pi) = \boxed{2\pi}$$

4 We will use cylindrical coordinates:

$$\vec{r}(\theta, z) = \langle \cos(\theta), \sin(\theta), z \rangle$$

$$\frac{\partial \vec{r}}{\partial \theta} = \langle -\sin(\theta), \cos(\theta), 0 \rangle, \quad \frac{\partial \vec{r}}{\partial z} = \langle 0, 0, 1 \rangle$$

$$\frac{\partial \vec{r}}{\partial \theta} \times \frac{\partial \vec{r}}{\partial z} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} \vec{i} & \vec{j} \\ -\sin\theta & \cos\theta \end{vmatrix}$$

$$= \langle \cos\theta, \sin\theta, 0 \rangle, \text{ so } \left\| \frac{\partial \vec{r}}{\partial \theta} \times \frac{\partial \vec{r}}{\partial z} \right\| = \sqrt{\cos^2\theta + \sin^2\theta} = 1.$$

Thus,

$$\iint_S x^2 \cancel{y}\, dS = \int_{\theta=0}^{2\pi} \int_{z=0}^{1} \cos^2\theta \, (1)\, dz\, d\theta$$

$$= \int_{\theta=0}^{2\pi} \cos^2\theta \left[\int_{z=0}^{1} dz \right] d\theta = \int_{\theta=0}^{2\pi} \cos^2\theta \, d\theta = \int_{\theta=0}^{2\pi} \frac{1}{2} + \frac{1}{2}\cos 2\theta \, d\theta$$

$$= \frac{1}{2}\left[\theta + \cos 2\theta\right]\Big|_{\theta=0}^{2\pi} = \boxed{\pi}$$