(1) (a) $$\int_{x=0}^{1}\int_{0}^{1}\frac{y}{(xy+1)^2}\,dy\,dx=\int_{y=0}^{1}\left\{\int_{x=0}^{1}\frac{y}{(xy+1)^2}\right\}dx\,dy \leftarrow \boxed{\begin{array}{l}u=xy+1\\ du=y\,dx\end{array}}$$

$$=\int_{y=0}^{1}\int_{u=1}^{1+y}\frac{du}{u^2}\,dy=\int_{y=0}^{1}\left[-\frac{1}{u}\right]\Big|_{u=1}^{1+y}dy$$

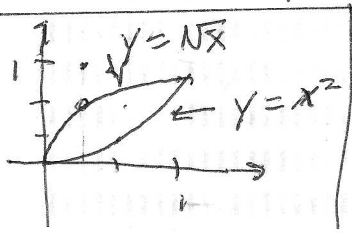
$$=\int_{y=0}^{1}\left(1-\frac{1}{1+y}\right)dy=\left[y-\ln(1+y)\right]\Big|_{y=0}^{1}=\boxed{1-\ln(2)}.$$

(b) $$\int_{1}^{2}\int_{0}^{y^2}e^{x/y^2}\,dx\,dy=\int_{y=1}^{2}\left[y^2e^{\frac{x}{y^2}}\right]\Big|_{x=0}^{y^2}dy$$

$$=\int_{y=1}^{2}e\,y^2-y^2\,dy=\int_{y=1}^{2}(e-1)\,y^2\,dy$$

$$=(e-1)\frac{y^3}{3}\Big|_{y=1}^{2}=\frac{(e-1)}{3}(8-1)=\boxed{\frac{7}{3}(e-1)}.$$

(2) (a)



(b) $$\int_{x=0}^{1}\int_{y=x^2}^{\sqrt{x}}(x+y)\,dy\,dx$$

(b) $\rightarrow$ $$=\int_{x=0}^{1}\left[xy+\frac{y^2}{2}\right]\Big|_{y=x^2}^{\sqrt{x}}dx=\int_{x=0}^{1}\left(x^{3/2}+\frac{x}{2}-x^3-\frac{x^4}{2}\right)dx$$

$$=\left[\frac{2}{5}x^{5/2}+\frac{1}{4}x^2-\frac{1}{4}x^4-\frac{1}{10}x^5\right]\Big|_{x=0}^{1}=\frac{2}{5}+\frac{1}{4}-\frac{1}{4}-\frac{1}{10}$$

$$=\boxed{\frac{3}{10}}$$

(3) $\dfrac{\partial \vec{r}}{\partial u} = \langle 1, 1, 2u \rangle$; $\dfrac{\partial \vec{r}}{\partial v} = \langle 0, 1, -4v \rangle$

$\dfrac{\partial \vec{r}}{\partial u}(1,1) = \langle 1, 1, 2 \rangle$; $\dfrac{\partial \vec{r}}{\partial v}(1,1) = \langle 0, 1, -4 \rangle$

$$\frac{\partial \vec{r}}{\partial u} \times \frac{\partial \vec{r}}{\partial v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 2 \\ 0 & 1 & -4 \end{vmatrix} = \vec{i} \begin{vmatrix} 1 & 2 \\ 1 & -4 \end{vmatrix} - \vec{j} \begin{vmatrix} 1 & 2 \\ 0 & -4 \end{vmatrix} + \vec{k} \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}$$

$$= \langle -6, 4, 1 \rangle,$$

so an equation for the tangent plane is:

$$\langle -6, 4, 1 \rangle \times \langle x-1, y-2, z+1 \rangle = 0,$$

or $-6(x-1) + 4(y-2) + (z+1) = 0$,

or $\boxed{-6x + 4y + z = 1}$

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(4) In spherical coordinates, the integral is:

$$\int_{\rho=0}^{2} \int_{\theta=0}^{2\pi} \int_{\varphi=0}^{\pi} \rho \, (\rho^2 \sin\varphi \, d\varphi \, d\theta \, d\rho)$$

$$= \int_{\rho=0}^{2} \rho^3 \int_{\theta=0}^{2\pi} \left\{ \int_{\varphi=0}^{\pi} \sin\varphi \, d\varphi \right\} d\theta \, d\rho$$

$$= \left\{ \int_{\varphi=0}^{\pi} \sin\varphi \, d\varphi \right\} \int_{\rho=0}^{2} \rho^3 \left\{ \int_{\theta=0}^{2\pi} d\theta \right\} d\rho$$

$$= \left\{ \int_{\theta=0}^{2\pi} d\theta \right\} \left\{ \int_{\varphi=0}^{\pi} \sin\varphi \, d\varphi \right\} \int_{\rho=0}^{2} \rho^3 \, d\rho$$

$$= (2\pi) \quad (2) \left[\frac{\rho^4}{4} \right] \Big|_{\rho=0}^{2}$$

$$= (2\pi)(2)[4] = \boxed{16\pi}$$
