Math. 302-01
Spring, 2005
R. B. Kearfott

Fifth Examination

Thursday, March 24, 2005

Instructions: This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly and explained. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully. You may keep this exam sheet. Each part of each is worth 30 points, and 10 points are "free".

1. Suppose the Earth's atmosphere extends from radius $\rho = 6,400$ kilometers to radius $\rho = 6,700$ kilometers, and the density of the atmosphere at height ρ kilometers is approximately
$$D(\rho) \approx 1.4 \times 10^9 e^{-0.105(\rho-6400)}$$
kilograms per cubic kilometer. From this, compute the approximate weight of the earth's atmosphere.
Hint: Integration by parts gives
$$\int u^2 e^{-c(u-a)} du = -\frac{1}{c} u^2 e^{-c(u-a)} - \frac{2}{c^2} u e^{-c(u-a)} - \frac{2}{c^3} e^{-c(u-a)}.$$
Also, you may take the upper limit of the Earth's atmosphere to be at $\rho = \infty$, if it is easier.

2. Consider
$$I = \int\int_{\mathcal{A}} \left(x^2 + y^2\right) dA,$$
where $\mathcal{A}$ is the triangle with vertices $(0,0)$, $(1,0)$, and $(1,1)$.
   (a) Compute I using rectangular coordinates.
   (b) Compute I using polar coordinates.
   Hint: the upper bound on r is a function of θ, that can be obtained by considering a trigonometric function, as in an example worked in class. Also,
$$\int \sec^4(\theta) d\theta = \frac{2}{3}\tan(\theta) + \frac{1}{3}\sec^2(\theta)\tan(\theta).$$